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The Single-Parametric Model of the Meshing by Cutting Cylindrical Gears Having Archimedean Spiral Curved Tooth Line

Márton MÁTÉ

Department of Mechanical Engineering,
Faculty of Technical and Human Sciences,
Sapientia Hungarian University of Transylvania, Tg. Mureş,
e-mail: mmate@ms.sapientia.ro

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Abstract: This paper presents a novel method of synthesis of cylindrical gears having opposite curvatures on the contacting flanks. The method is an improved variant of cylindrical gears having Archimedean spiral curved toothline – developed by the author [3, 4]. Due to the increased cutting capacity of the milling head used in the technological variant that will be presented the kinematics of the meshing results more sophisticated like in the simplest case where a single radial feed is applied. Due to the fact that the present technology uses a cutter head with three cutter groups that executes a tangential feed motion the meshing process can be discussed in three variants: single parameter meshing, bi-parametric meshing and a novel method developed by the author – the double meshing model. This paper discusses the most widely used method in the theory of gear meshing, the single parametric method.

The model of the improved cutting tool, the kinematics of meshing, and the mathematical model of gearing are presented in detail.

Keywords: cylindrical gear, curved toothline, spiral, gearing, meshing.

1. Why cylindrical gears with Archimedean spiral shaped toothline?

The improvement of the contact between the tooth flanks and the increase of the load capacity remains a permanent challenge in the world of gear science. The development of cylindrical gears with curved teeth represents one of the possible ways in this direction. Cylindrical gears represent a class of machine elements present in most mechanical power transmission applications. In order to improve their load capacity by given dimensions there exist two tasks that must be fulfilled: the modification of the tooth profile consisting in addendum chamfering and sometimes in admissible dedendum undercut, and the localization of the theoretical contact in a point instead of the theoretical line –

modification defined as localization of the contact patch. This last task is very important to be fulfilled in case of high performance gears of all types. In the case of cylindrical gears there exist two basic types assisted by well suited and robust manufacturing technologies: right teethed and helical teethed cylindrical gears. The localization of the contact patch is realized here by applying high performance but expensive grinding or shaving technologies. By external cylindrical gears –both spur or helical – always convex tooth-surfaces contact.

It is mathematically proved by Hertz's contact theory that better load capacity is reached if the contact occurs between surfaces having opposite curvatures. The idea was applied for the beginning by the bevel gears of automotive power transmission where the flank line of the teeth is a looped epicycloid.

The most famous development regarding the improvement of the load capacity of a cylindrical gear pair is known as the Wildhaber-Novikov teething. This is realized using a set of complementary racks with circular arc shaped tooth profile. As a consequence one of the gears results with convex while the other with concave tooth profile. Despite of the thorough study and research (realized almost in the '60-s) the achieved results weren't reach the expected parameters. It was also remarked that the form and the position of the contact patch modifies intensely with the modification of the axis distance [1], [2].

In order to avoid the disadvantage occurred by Wildhaber-Novikov gear teeth the sense of curvature should be – in the vision of the author – set along the tooth length. In this case the generating element of the gear pair is a self-moving rack with Archimedean spiral curved toothline. The kinematic geometry of the gear pair meshing fulfills Olivier's second principle as it is shown in the right sketch of *Fig. 1*. This represents a plain section through the axis of the milling head. The left sketch of *Fig. 1* present the structure of the milling head. A defined number of profiled cutters are suited on an Archimedean spiral whose pitch is equal to the pitch of the generating rack. The profile of each cutter is corresponding to the profile of the rack defined by DIN 3972. These plain trapezoidal symmetric profiles are included in axial planes of the milling head. As a consequence of the cutter placement defined before, if the milling head rotates, a sliding rack profile appears in the axial plane that slides along a radial direction. If two gears are positioned correspondently to the moving rack, the same geometric dependencies will be reached as in the case of spur gears. However, in parallel sections there eexist some delays that lead to the curvature of the generating tooth and finally to the curving of the generated toothline. Finally, the effect consists in the cutting of tooth spaces limited by a concave and a convex surface.

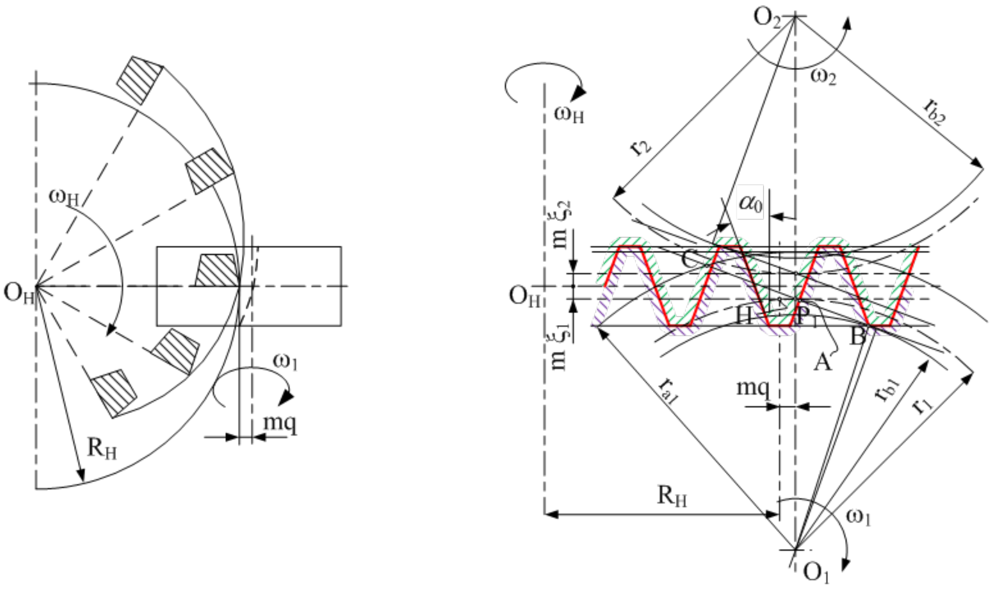

Figure 1: The sketch of the rack generating milling head and the coupling gear pair.

It is proven [3] that one single milling head body is sufficient for the generating of both elements of the gear pair. The problem consists in the changing of the sense of the leading spiral by positioning the cutters on the opposite side as shown in *Fig. 2*. Due to this a head with symmetric cutter positioning slots solves the problem. This will constitute another advantage versus the Wildhaber-Novikov variant where two distinct cutting tools are necessary to generate the gear-pair. The curvatures on the coupling flanks can be easily adjusted using the tangential shifting parameters q_1, q_2. The localization of the contact and the relative sliding of the coupling surfaces can be optimized by the rational selection of the profile shifting parameters ξ_1, ξ_2.

2. The principle of meshing using tangential feed

The gear meshing kinematics presented above uses the radial feed to achieve the complete depth of cut. The cutting process based on this is estimated to be slow. In order to increase the productivity, radial feed will be replaced with tangential feed, and the structure of the milling head will be improved, by considering Z_0 groups of cutters, each of them containing 3 up to 5 cutters.

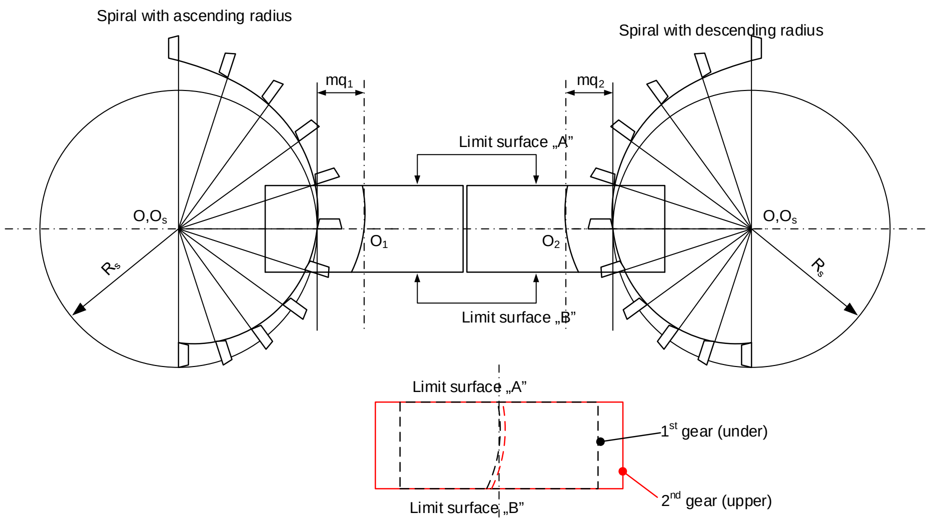

Figure 2: The meshing of the elements of the gear pair using a single cutter head with symmetrical positioning of the inserts.

The sketch of the cutter head and the principle of work is presented in *Fig. 3*. The cutter groups angularly equidistant positioned in the cutter head. Here cutters with a single edge are used, each of them meshing one surface of the generating rack tooth or gap. The cutters are fitted on equiangular distanced Archimedean spirals with the spiral pitch given by

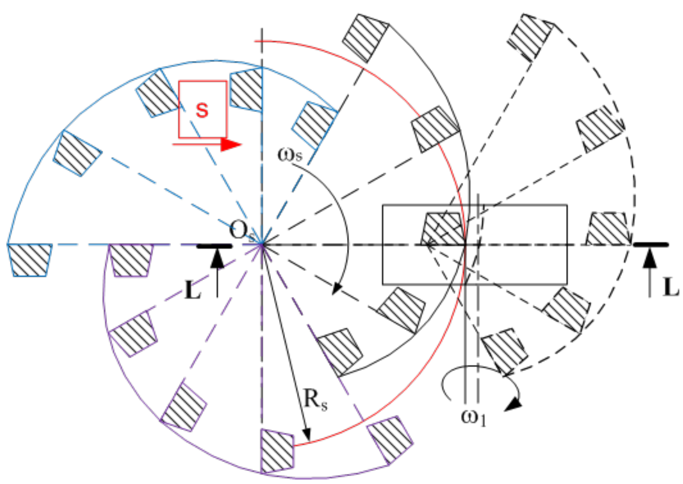

Figure 3: The cutter head and the principle of tangential feed.

$$p_{sp} = Z_0 \frac{\pi m}{2\pi} = Z_0 \frac{m}{2} \tag{1}$$

The magnitude of teeth's curvature is the reference radius R_s of the spiral.

The reference edge is the central edge e.g. the 3[th] in each group. Denoting with τ_s the angular distance between two consecutive edges, the limit radius values of the leading spiral are

$$R_s - p_{sp} \frac{z_s - 1}{2} \tau_s \le \rho \le R_s + p_{sp} \frac{z_s - 1}{2} \tau_s \tag{2}$$

The kinematics of the meshing is shown on *Fig. 4*. A number of 3 frames are used as follows: frame $(x_0 y_0 z_0)$ of the machine body, considered stationary; frame $(x_s y_s z_s)$ of the milling head; finally, frame $(x_1 y_1 z_1)$ bound to the machined gear. The chosen leading parameter in this kinematic model is the rotation angle of the milling head about its own axis φ_s.

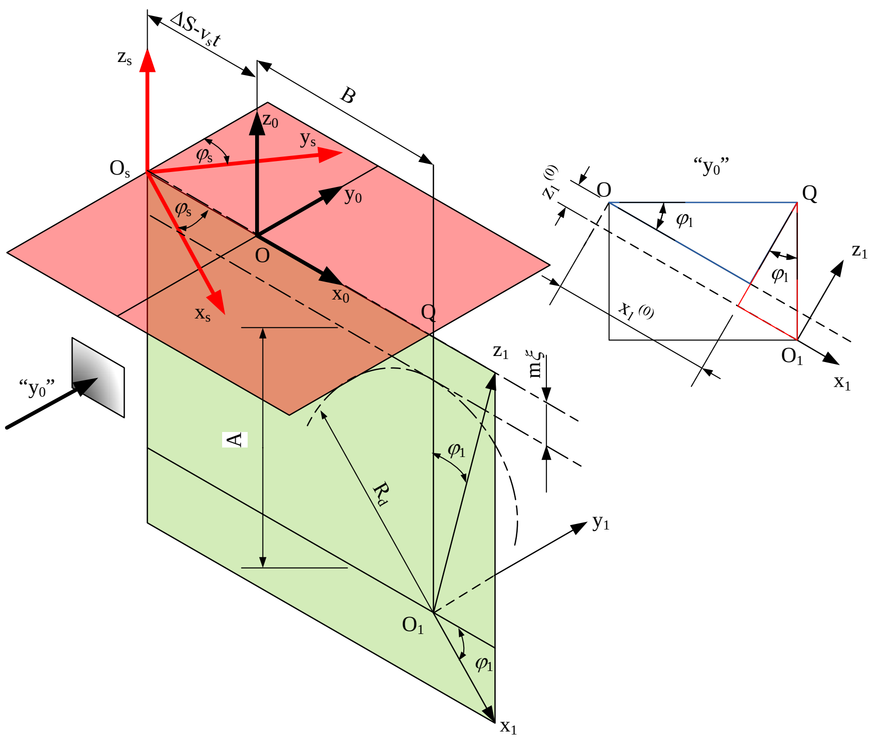

Figure 4: The kinematics of meshing.

The tangential feed can be defined considering the tangential velocity $\mathbf{v}_s$ or in dependence with the rotation angle φ_s, meaning in this case the length of tangential sliding. This last definition is more adequate for the next purposes. Finally, the rotation angle φ_1 of the machined gear depends on the rotation angle of the milling head and the corresponding tangential displacement of the milling head $\Delta x(\varphi_s)$. The geometric condition is that of slideless rolling of the pitch circle on the pitch line of the imaginary rack.

In order to prime the function $\Delta x(\varphi_s)$ let's suppose that cutting velocity v_c and tangential feed length pro minute s_1 are known or determined with analogy to other similar cutting processes. In this case, the rotation *n* of the milling head can be computed with the well-known formula

$$n = \frac{10^3 v_c}{2\pi R_s} \tag{3}$$

Considering that the tangential displacement during one complete rotation amounts s_1 / n the sought dependence can be primed as

$$\Delta x = \frac{s_1}{n}\frac{\varphi_s}{2\pi} = \frac{R_s s_1}{10^3 v_c}\varphi_s = \psi \varphi_s \tag{4}$$

The rotation of the machined gear, considering equation (4) and the pitch of the spiral given by (1) became

$$\varphi_1 = \frac{\varphi_s p_{sp} + \psi\varphi_s}{R_d} = \frac{p_{sp} + \psi}{R_d}\varphi_s \tag{5}$$

Using dependences (4) and (5) the model of the single parametric meshing can be built up.

3. The equations of the single parametric mesching

A. The equations of the generating surfaces

Generating surfaces in the theory of meshing are considered – both one and two parametric cases – the support surfaces of the cutting edges, which can take very variate forms; their definition stats from the presumption that the cutting tool possesses an infinity of edges e.g. a grinding wheel [5], [6], [7]. In the case described in this paper the generating surfaces are the support of the cutting edges. Generatrix is the straight segment of cutting edge while the directory is the Archimedean spiral. The equations can be easily written using *Fig. 5*.

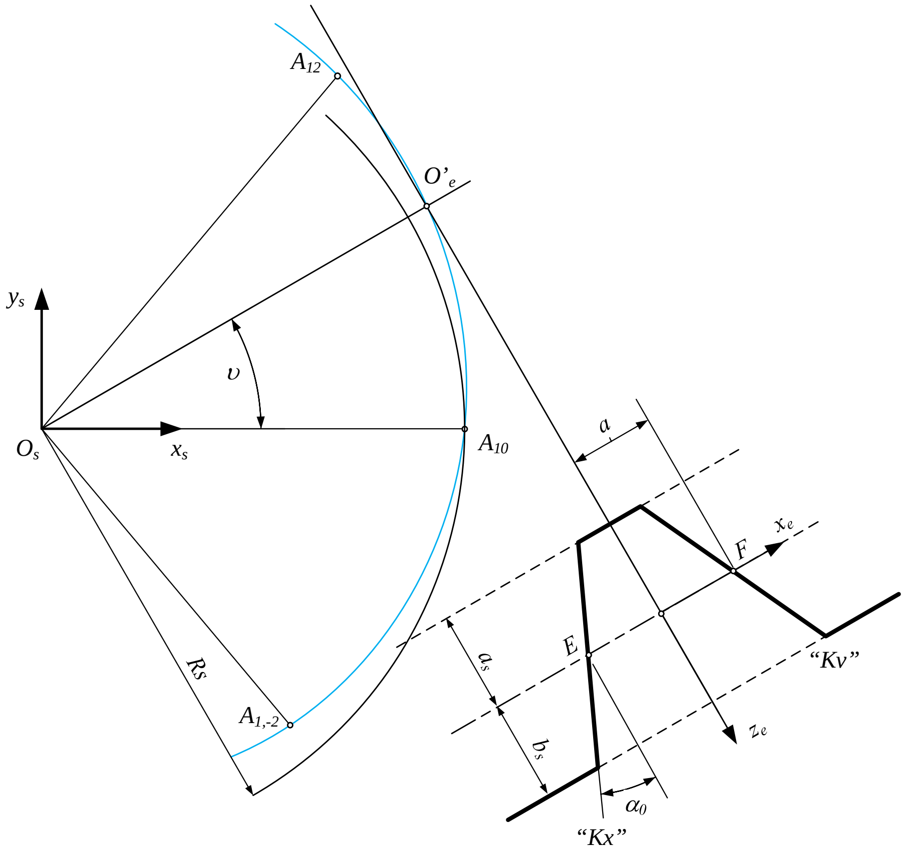

Figure 5: The built-up of the generating surfaces.

Here is to remark that generating surfaces are meshed by the cutting edges when frame $(x_e y_e z_e)$ moves counterclockwise with origin O_e on the Archimedean spiral while axis $O_s z_s$ is still contained in plane $(x_e z_e)$. Notation "*Kx*" denotes the convex toothspace generating edge. By analogy, notation "*Kx*" is used for the concave surface. Using the matrix transformation between the involved frames

$$\mathbf{r}_s = \mathbf{M}_{se}\,\mathbf{r}_e \Leftrightarrow \begin{pmatrix} x_s \\ y_s \\ z_s \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\upsilon & -\sin\upsilon & 0 & (R_s + p_{sp}\upsilon)\cos\upsilon \\ \sin\upsilon & \cos\upsilon & 0 & (R_s + p_{sp}\upsilon)\sin\upsilon \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_e \\ y_e \\ z_e \\ 1 \end{pmatrix} \tag{6}$$

the generalized equations of the generating surfaces result as follows:

$$\begin{cases} x_s(u,\upsilon) = \left(i\left(a + u\,\mathrm{tg}\,\alpha_0\right) + R_s + p_{sp}\upsilon\right)\cos\upsilon \\ y_s(u,\upsilon) = \left(i\left(a + u\,\mathrm{tg}\,\alpha_0\right) + R_s + p_{sp}\upsilon\right)\sin\upsilon, \quad i \in \{-1;1\} \\ z_s(u,\upsilon) = u \end{cases} \tag{7}$$

B. The equation of gearing

The equation of gearing will be written using the kinematic model developed by Litvin [8]. First the equations of the generating surface families must be written. This is accessible using geometric dependences emphasized on *Fig. 4*. The equation of the transformation is

$$\mathbf{r}_1 = \mathbf{M}_{1s}\,\mathbf{r}_s \tag{8}$$

where the transformation matrix has the following expression:

$$\mathbf{M}_{1s} = \begin{pmatrix} \cos\varphi_1\cos\varphi_s & \cos\varphi_1\sin\varphi_s & -\sin\varphi_s & -(B + \Delta s - \psi\varphi_s)\cos\varphi_1 - A\sin\varphi_1 \\ -\sin\varphi_s & \cos\varphi_s & 0 & 0 \\ \sin\varphi_1\cos\varphi_s & \sin\varphi_1\sin\varphi_s & \cos\varphi_1 & -(B + \Delta s - \psi\varphi_s)\cos\varphi_1 + A\sin\varphi_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

The relative velocity can be written with help of *Fig. 6*. First, the tangential velocity component of the milling head is

$$\dot{\mathbf{s}} = \psi\omega_s\,\mathbf{i}_0 \tag{10}$$

Adding this to the component resulted from the superposition of rotations the relative velocity can be computed as follows:

$$\begin{aligned} \mathbf{v}_{O_s}^{(s,1)} &= \mathbf{v}_{O_s}^{(s)} - \mathbf{v}_{O_s}^{(1)} = \psi\omega_s\,\mathbf{i}_0 + \boldsymbol{\omega}_{O_s}^{(s)} \times \mathbf{r}_s - \boldsymbol{\omega}_{O_1}^{(1)} \times \mathbf{r}_1 = \\ &= \psi\omega_s\left(\cos\varphi_s\mathbf{i}_s + \sin\varphi_s\mathbf{j}_s\right) + \left(\boldsymbol{\omega}_{O_s}^{(s)} - \boldsymbol{\omega}_{O_s}^{(1)}\right) \times \mathbf{r}_s - \mathbf{A}_w \times \boldsymbol{\omega}_{O_s}^{(1)} \end{aligned} \tag{11}$$

The normal vectors of the generating surfaces are computed using the classical formula $\mathbf{n} = \mathbf{r}'_u \times \mathbf{r}'_\upsilon$. With all partial results, after a long calculus, equation of gearing can be transformed in an algebraic equation of 2nd degree in parameter u:

$$a'u^2 + 2k'u + c' = 0 \qquad (12)$$

The coefficients of the equation have the following expressions:

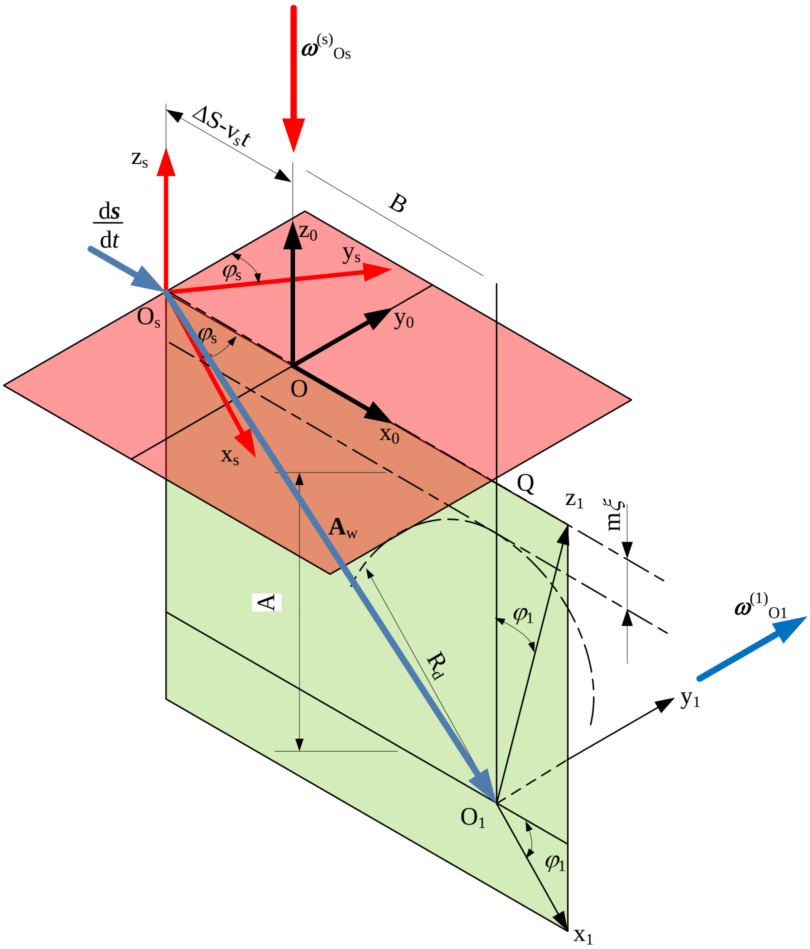

Figure 6: The relative velocity.

$$a' = \frac{-\operatorname{tg}\alpha_0 \cos(\varphi_s - \upsilon)}{\cos^2 \alpha_0}$$

$$\begin{aligned} 2k' = {} & p_{sp} \sin(\varphi_s - \upsilon) - \left[(R_s + a + p_{sp}\upsilon)(1 + 2\operatorname{tg}^2 \alpha_0) + A \operatorname{tg}\alpha_0 \right] \cos(\varphi_s - \upsilon) + \\ & + i_{s1} p_{sp} tg\alpha_0 + (B + \Delta S - \psi\varphi_s) \operatorname{tg}^2 \alpha_0 \\ c' = {} & A\, p_{sp} \sin(\varphi_s - \upsilon) - (p_{sp}\upsilon + R_s + a)\left[A + (p_{sp}\upsilon + R_s + a) \operatorname{tg}\alpha_0 \right] \cos(\varphi_s - \upsilon) + \\ & + \left[i_{si}\, p_{sp} + \operatorname{tg}\alpha_0 (B + \Delta S - \psi\varphi_s) \right] (p_{sp}\upsilon + R_s + a) \\ i_{s1} = {} & \frac{1}{i_{1s}} = \frac{R_d}{p_{sp} + \psi} \end{aligned} \tag{13}$$

C. The computing of the meshed surface

The equation of gearing returns solutions of type

$$u_i = f_i(\upsilon, \varphi_s), \quad i = \overline{1,2} \tag{14}$$

for both toothspace limiting surfaces. Due to the fact that equation of gearing returns *all* solutions, it is necessary to separate only those that contribute to build up the real materialized part of the machined gear. As a conclusion, some parameter limiting conditions must be defined.

If the width of the machined gear is B_k then *y* coordinate of the generating surfaces must fulfill the condition:

$$-0.5B_k \le y_1 \le 0.5B_k \tag{15}$$

Considering the start position of the frames involved in the mathematical model, it can be easily established the limits of the rolling angle φ_1 considering an analogy with the coupling of a spur involute gear and a rack. After some calculus it results

$$\varphi_{11} = -\frac{\left(\sqrt{r_a^2 - r_b^2} - \sqrt{r_a^2 - R_d^2}\right)\dfrac{1}{\cos\alpha_0} + (a - m\xi \operatorname{tg}\alpha_0)}{R_d} \tag{16}$$

$$\varphi_{12} = \frac{\dfrac{h_{0a}^* + c_a^* - m\xi}{\sin\alpha_0 \cos\alpha_0} - (a - m\xi \operatorname{tg}\alpha_0)}{R_d} \tag{17}$$

Finally, the value of the edge parameter moves within the interval $u \in [-a_s, b_s]$, while the spiral parameter $\upsilon \in [-\pi / Z_0, \pi / Z_0]$.

Considering two sets of N_i respectively N_j equidistant values on the intervals mentioned before, the generating surface can be approximated as a system of nodes $\left(x_1^{(i,j)}, y_1^{(i,j)}, z_1^{(i,j)}\right)$. Now the equation of gearing will be brought on the following particular form:

$$u_i - f\left(\upsilon_j, \frac{R_d}{p_{sp} + \psi} \varphi_1^{(i,j)}\right) = 0 \tag{18}$$

The angle $\varphi_1^{(i,j)}$ results numerically for each (u_i, υ_j) parameter combination. If the solution meets the interval defined by the limit values given through expressions (16) and (17) a real point results on the meshed surface.

Another possible way to compute the points of the meshed surface consists in the determination of the contact curve on the generating surface. In this case, the gear's rolling parameter φ_1 will be represented as a set of N_i values on the rolling interval. For each φ_1^i and each υ_j a correspondent value $u_{i,j}$ is obtained through solving equation (12). With this, the contact curve on the generating surfaces are written using equations (7). The meshing surface results with the transposing of the contact curve in the frame (x_1, y_1, z_1) using the matrix transformation (8).

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Unspecified notations

α_0 – the rack profile angle
m – the module
R_d – the radius of the pitch circle
r_a – the radius of the addendum circle
r_f – the radius of the dedendum circle
r_b – the radius of the involute basic circle
a – the half width of the generating rack profile on the pitch line

ξ – profile shifting coefficient

$\boldsymbol{\omega}_s$ – angular velocity vector of the milling head

$\boldsymbol{\omega}_1$ – angular velocity vector of the machined gear

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