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Solution concepts for coevolutionary 2-period cumulated games

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Abstract. Based on strategic games, a new type of dynamic game has been introduced, the n-period cumulated game (n-PCG), where players engage in a repetitive play of a constituent strategic game for n number of times (an accumulation period), without receiving their payoff after each stage of the game, but only the cumulated payoffs of all the stages of the game at the end of the accumulation period. Some solution concepts for n-PCGs are discussed, namely subgame perfect equilibria and Nash equilibria. Then a two-population based genetic algorithm is introduced in order to find these equilibria in 2-period cumulated games.

1 Introduction

Genetic algorithms are a well-known optimization method introduced by John Holland in the early 1970s [3]. Nash strategy [4, 5] is the most commonly encountered solution concept in game theory. The idea to use genetic algorithms together with the Nash strategy concept, such that the algorithm searches for the Nash equilibrium, belongs to Sefrioui [10]. As described in [7] at each generation a player improves its strategy with respect to the other players' best strategies of the previous generation: Nash equilibrium is reached when no player can improve its strategy. Based on this, NCA (Nash Coevolution

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Algorithm), a two population based genetic algorithm, finds subgame perfect equilibria in 2-period cumulated games interpreted as extensive games with imperfect information.

2 n-period cumulated game (n-PCG)

The notion of cumulated game basically means that, for instance, having two individuals playing a number of strategic games, there exists a mechanism that sums and withholds the benefits until the players have completed n plays of the strategic game; n is called the length of the accumulation period. Their accumulated benefits are reported only after n stage games. Our goal is to model the situations where players engage in games, in which they have different knowledge of the previous plays, and in which they receive their payoff after different accumulation periods. For example, consider the model of the relation between an employer and an employee where the employer agrees to pay the employee a small amount of money every two weeks even though he may gain benefits from the employee's work at every two months. The accumulation period of the employee is two weeks and the accumulation period of the employer is two months. However, here the focus is on the notion of n-period cumulated games (i.e. equal accumulation periods), and on the analysis of four models of 2-PCGs distinct by their elementary game.

The notion of cumulative benefit game already appeared in [11]. There the author argued that in the context of repetitive games and a strong temporal discounting, accumulation can promote a cooperative strategy. However, an n-PCG is a not a repetitive game even though it is a dynamic one.

2.1 n-period cumulated games as strategic games

An n-PCG can be described as a strategic game. Take for example the strategic game in Fig. 1(a) and an accumulation period of two, i.e., after the first stage of the game no payoff is reported, and after the second stage of the game the players receive their cumulated payoffs over the two plays.

This 2-PCG can be viewed as a strategic game, as depicted in Table 1. In this game the pure strategies for Player 1 *(AA, AB, BA , BB)* and Player 2 *(CC, CD, DC, DD)* are composed by the pure strategies of the initial strategic game. The payoffs are the cumulated payoffs over a period of the cumulated game. For example, if the first player (the row player) chooses AB and the second player chooses DD, then the payoff is 2 for the first player and 6 for the second player, i.e., in the first stage of the game they play AD, which has an unreported payoff of $(1, 5)$, and in the second stage they play *BD*, which has an unreported payoff of $(1, 1)$, thus having an accumulated reported payoff of $(2, 6)$.

	CC	CD	DC	DD
AA	6,6	4,8	4,8	2,10
AB	8,4	4,4	6,6	*2,6*
BA	8,4	6,6	4,4	2,6
BB	10,2	6,2	6,2	2,2

Table 1: A 2-PCG based on the strategic game from Fig. 1(a)

2.2 n-period cumulated games as dynamic games

An image of the described game is that of a dynamic game. A dynamic game captures the idea that players act sequentially and can incorporate information about earlier moves in the game in choosing their next move. Even though a stage game (called constituent game) is being repeatedly played, the difference between an n-Period Cumulated Game and a finitely repeated game is that the benefit/payoff is reported only after an accumulation period.

The n-PCG can be represented as an extensive game. For instance, the static game in Fig. 1(a) is equivalent to the dynamic game represented in Fig. 1(b). The dashed line between some nodes means that the current decision maker does not know in which state she is at. Therefore, the extensive game can be viewed as a 2×2 strategic game where players act simultaneously. This is the case of extensive games with imperfect information, i.e., each player, when making a decision, is not perfectly informed about the events that have previously occurred [9, 6].

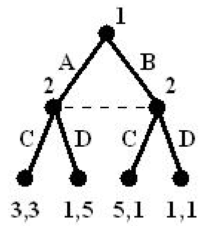
3 Solution concepts for n-period cumulated games

Interpreting the n-period cumulated game as a new strategic game, the apropriate solution concept is that of Nash equilibrium [4, 5], the most commonly encountered solution concept in game theory. The Nash equilibrium and most of its variants express the idea that each player individually maximizes its utility [1].

Interpreting the n-period cumulated game as an extensive game, the adequate solution concept is that of subgame perfect equilibrium. A subgame is

	C	D
A	3,3	1,5
B	5,1	1,1

(a) the game in its normal form



(b) the game in its extensive form

Figure 1: Views of the same 2×2 strategic game

a subtree from a game's directed tree that has the properties: it begins at a decision node, it gives the initial player all the decisions that have been made until that time, and it contains all the decision nodes that follow the initial node. The subgame perfect equilibrium (or subgame perfect Nash equilibrium) is a refinement of Nash equilibrium (NE) that induces a NE in every subgame (subtree) of that game. In an $\mathfrak{n}$-PCG there are subgames with simultaneous decisions, therefore all possible Nash equilibria of that subgames may appear in a subgame perfect Nash equilibrium.

3.1 Numerical experiments

After excluding the games that are strategically trivial in the sense of having equilibrium points that are uniquely Pareto-efficient, there remain four archetypal 2×2 games: prisoner's dilemma, chicken (Hawk-Dove), battle of the sexes, and leader [8]. These games where taken as constituent games for 2-PCGs. The theoretical solving of these 2-PCGs is described by solving the 2-PCG based on the Hawk-Dove Game.

3.1.1 Hawk-Dove game

The strategic form of this game depicted in Table 2 has two Nash equilibria: *DH* and *HD*. The game tree from Fig. 2 represents the 2-PCG based on this Hawk-Dove game. It has five subgames, four of them are the extensive form of the strategic constituent game (Table 2), and one is the whole tree. Working through backwards induction, the four subgame perfect equilibria are found. These are the dotted paths from root to leaves in Fig. 2.

An $\mathfrak{n}$-PCG can be described as a strategic game, hence the 2-PCG depicted in Fig. 2 can be represented as the strategic game in Table 3. Using the best response strategy, the Nash equilibria of this strategic game are found. They are the same with those of the subgame perfect equilibria found before. Certainly, this is due to the manner in which the game was constructed, but nevertheless, this gives us a static insight of a dynamic process.

	D	H
D	3,3	*1,5*
H	*5,1*	0,0

Table 2: The Hawk-Dove game

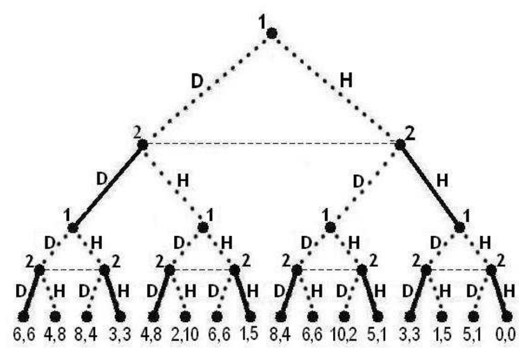


Figure 2: Game tree for a 2-PCG based on the Hawk-Dove game from Table 2. The Nash equilibria of every subgame are marked with dots. The dotted paths from root to leaves are the subgame perfect equilibria

	CC	CD	DC	DD
AA	6,6	4,8	4,8	*2,10*
AB	8,4	3,3	*6,6*	1,5
BA	8,4	*6,6*	3,3	1,5
BB	*10,2*	5,1	5,1	0,0

Table 3: A 2-PCG based on the strategic game from Table 2

Similar with the Hawk-Dove game based 2-PCG the solutions for the 2-PCGs with the constituent strategic games prisoner's dilemma, battle of the sexes and leader are found.

4 NCA (Nash coevolution algorithm)

A coevolution algorithm for finding subgame perfect equilibria in 2-period cumulated games is proposed. This algorithm is called Nash coevolution algorithm (NCA).

4.1 Encoding a strategy

With NCA, a chromosome represents a strategy that is a function of the state of the game: every possible state (history) has one slot $i, 0 \leq i < s$, in the s^m string that codes the move the player will take if she uses that strategy and she encounters the history with index i; s is the number of states of the constituent strategic game (stage game), m is the length of the history (the number of stage games that the player recalls) and the hypothetical game is there to induce the first actions that the player will take. The difference from [2] is that here there is no restriction for m.

4.2 Fitness assignment

Every player is represented by a population, which maximizes the player's payoff at every generation. The populations evolve by optimizing at each step their chromosomes using the other population's best q chromosomes from the previous generation [7], i.e. each player has a population that tries to maximize its payoff using the best solutions found by the other player's population one generation before. The fitness of each chromosome will be the mean payoff it receives after playing with each of the other player's best chromosomes from the previous generation.

In games with imperfect information the notion of subgame perfect equilibrium may require mixed strategies, for instance the game of matching pennies. This implementation does not support these types of strategies.

5 Convergence, stability and more

NCA was tested using fixed and variable parameters. The fixed one where:

- *population size = 40* strategies for each player;
- a number of *700 to 1000 generations* for each run;
- *recombination probability = 0.2%*;
- *accumulation period = 2* for both players.

The other parameters were:

- *exchange* $\in \{5, 20, 40\}$, where *exchange* is the number of individuals used to evaluate a strategy;
- *probability of mutation, p.m.* $\in \{0.1\%, 0.2\%, 0.3\%\}$;
- *history length* $\in \{1, 2\}$.

They varied one at a time, thus allowing an objective impact analysis.

Different parameters configuration gave an insight into the stability of NCA and proved empirically that

$$stability \approx \frac{1}{exchange \times p.m.}.$$

The convergence time to an equilibrium is between 40 and 60 generations for all the configurations of the Leader game. For the Battle of sexes, for the test configurations 1 to 9 (*history length of 1*) an equilibrium was found in less than 70 generations in more than 90% of the cases; for the last three test configurations (*history length of 2*), the convergence time to an equilibrium is under 40 generations. For the Prisoner's dilemma game, it needs less than 30 generations to achieve its equilibrium in all the configurations. For the Hawk-Dove game the algorithm achieves an equilibrium under 70 generations with a probability of 87%.

6 Conclusion and future work

Based on strategic and repetitive games, a new dynamic game called n-period cumulated game has been introduced. Solution concepts for these n-PCGs, namely subgame perfect equilibria and Nash equilibria, were analyzed. Then 2-PCGs were tested in a coevolutionary environment (provided by the developed NCA) based on archetypal 2×2 games. The main result was that players eventually play a Nash equilibrium in every subgame, thus a subgame perfect equilibrium, even though they are not aware of their payoff until a certain number of stage games are played.

An interesting focus for future work is that of working with different levels of accumulation periods, testing the outcome of having the players engage in these basically different games. Another challenging issue can be that of letting two players play a number of games between themselves, thus transforming the n-PCG into a repetitive one; for instance, preliminary tests show that in the repetitive 2-PCG based on the prisoner's dilemma the players start by cooperating.

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