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On degree sets in k-partite graphs

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Abstract. The degree set of a k-partite graph is the set of distinct degrees of its vertices. We prove that every set of non-negative integers is a degree set of some k-partite graph.

1 Introduction

In a graph G, the degree of a vertex v_i, denoted by d_{v_i} (or simply d_i), is the number of edges which are incident on v_i. A sequence of non-negative integers $[d_1, d_2, \ldots, d_p]$ is called the degree sequence of a graph G if the vertices of G can be labelled $v_1,\ v_2, \ldots, v_p$ such that $deg\ v_i = d_i$ for each i, $1 \leq i \leq p$. The terminology and notations used in this paper are same as in [6, 25].

The set of distinct degrees of the vertices of a graph is called its degree set. The following result can be found in [8].

Theorem 1 [8] *Any set* D *of distinct positive integers is the degree set of a connected graph and the maximum order of such a graph is* $M + 1$*, where* M *is the maximum integer in the set* D.

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More on degree sets in graphs can be seen in [1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 29, 30, 31]. Analogous results in directed graphs and signed graphs can be found in [17, 18, 19, 20, 21, 22, 23, 24, 26, 27].

2 Degree sets in k-partite graphs

A k-partite graph ($k \geq 2$) is a graph G whose vertex set can be partitioned into k nonempty disjoint sets $V_1, V_2, \ldots, V_k$, known as partite sets, such that $v_i v_j$ is an edge of G if v_i is in some V_i and v_j is in some V_j ($i \neq j$). A k-partite graph with partite sets $V_1, V_2, \ldots, V_k$ is denoted by $G(V_1, V_2, \ldots, V_k)$. For $k = 2$ and $K = 3$ we get respectively bipartite graph and 3-partite graph. Also, a k-partite graph $G(V_1, V_2, \ldots, V_k)$ is said to be connected if each vertex $v_i \in V_i$ is connected to every vertex $v_j \in V_j$ ($i \neq j$). The degree of a vertex v_i in a k-partite graph $G(V_1, V_2, \ldots, V_k)$ is the number of edges of $G(V_1, V_2, \ldots, V_k)$ which are incident to v_i and is denoted by d_{v_i} or d_i. Let $G(V_1, V_2, \ldots, V_k)$ be a k-partite graph with $V_i = \{v_{i1}, v_{i2}, \ldots, v_{ip_i}\}$, $1 \leq i \leq k$ and let $d_{i1}, d_{i2}, \ldots, d_{ip_i}$ be the respective degrees of $v_{i1}, v_{i2}, \ldots, v_{ip_i}$. Then the sequence $D_i = [d_{i1}, d_{i2}, \ldots, d_{ip_i}]$, $1 \leq i \leq k$, are called degree sequences of $G(V_1, V_2, \ldots, V_k)$.

The set of distinct degrees of the vertices of a k-partite graph $G(V_1, V_2, \ldots, V_k)$ is called its degree set.

The following result is given in [16].

Theorem 2 [16] *Every set of positive integers is a degree set of some connected bipartite graph.*

The following result can be seen in [7].

Theorem 3 [7] *Every set of positive integers, except* $\{1\}$ *is a degree set of some connected* 3*-partite graph.*

Now, we have the following observation.

Theorem 4 *Every singleton set of positive integers is a degree set of some* k*-partite graph.*

Proof. Let $D = \{d\}$, where d is a positive integer. For $d = 1$, construct a k-partite graph $G(V_1, V_2, \ldots, V_k)$ as follows

$$\begin{aligned} V_1 &= A_{11} \cup A_{12} \cup \cdots \cup A_{1(k-2)} \cup A_{1(k-1)}, \\ V_2 &= A_{21}, \\ V_3 &= A_{31}, \\ &\vdots \\ V_{k-1} &= A_{(k-1)1}, \\ V_k &= A_{k1}, \end{aligned}$$

with $A_{1p} \cap A_{1q} = \emptyset$ $(p \neq q)$, $|A_{ij}| = 1$ for all i, j where $1 \leq i \leq k$, $1 \leq j \leq k-1$. Let there be an edge from the vertex of $A_{1(i-1)}$ to the vertex of A_{i1}. Then the degrees of the vertices of $G(V_1, V_2, \ldots, V_k)$ are as follow.

For $2 \leq i \leq k$

$$d_{a_{1(i-1)}} = d_{a_{i-1}} = |A_{1(i-1)}| = 1 = d, \text{ for all } a_{1(i-1)} \in A_{1(i-1)},\ a_{i1} \in A_{i1}.$$

Therefore, degree set of $G(V_1, V_2, \ldots, V_k)$ is $D = \{d\}$.

Now we assume that $d \geq 2$. For $k = 2$, consider the bipartite graph $G(V_1, V_2)$ with $|V_1| = |V_2| = d$ and let there be an edge from each vertex of V_1 to every vertex of V_2. Then the degrees of the vertices of $G(V_1, V_2)$ are as follows.

$$d_{v_1} = d_{v_2} = |V_1| = d, \text{for all } v_1 \in V_1,\ v_2 \in V_2.$$

Therefore, degrees set of $G(V_1, V_2)$ is $D = \{d\}$.

If $k \geq 3$ is odd, say $k = 2m + 1$ where $m \geq 1$, construct a $2m + 1$-partite graph $G(V_1, V_2, \ldots, V_{2m+1})$ as follows.

Let $V_1 = A_1$, $V_2 = A_2 \cup B_2$, $V_3 = A_3, \ldots, V_{2m} = A_{2m}$, $V_{2m+1} = A_{2m+1}$ with $|A_i| = |B_2| = d - 1$ for all i, $1 \leq i \leq 2m + 1$, $A_2 \cap B_2 = \emptyset$. Let there be an edge **(i)** from each vertex of A_i to every vertex of A_{i+1} for all odd i, **(ii)** from distinct vertices of A_i to distinct vertices of A_{i+1} for all even i, **(iii)** from distinct vertices of A_1 to distinct vertices of B_2, and **(iv)** from each vertex of A_{2m+1} to every vertex of B_2. Then the degrees of the vertices of $G(V_1, V_2, \ldots, V_{2m+1})$ are as follows.

For $1 \leq i \leq 2m + 1$

$$d_{a_i} = d_{b_2} = |A_i| + 1 = d - 1 + 1 = d \text{ for all } a_i \in A_i \text{ and } b_2 \in B_2.$$

Therefore, degree set of $G(V_1, V_2, \ldots, V_{2m+1})$ is $\{d\}$.

Again, if $k \geq 4$ is even, say $k = 2m + 2$ where $m \geq 1$, consider a $2m + 2$-partite graph $G(V_1, V_2, \ldots, V_{2m+2})$ with $|V_i| = d - 1$ for all i, $1 \leq i \leq 2m + 2$.

Let there be an edge **(i)** from each vertex of V_i to every vertex of V_{i+1} for all odd i, **(ii)** from distinct vertices of V_i to distinct vertices of V_{i+1} for all odd i, and **(iii)** from distinct vertices of V_1 to distinct vertices of V_{2m+2}. Then the degrees of the vertices of $G(V_1, V_2, \dots, V_{2m+2})$ are as follows

For, $1 \leq i \leq 2m+2$,

$$d_{v_i} = |V_i| + 1 = d - 1 + 1 = d, \text{ for all } v_i \in V_i.$$

Therefore, degree set of $G(V_1, V_2, \dots, V_{2m+2})$ is $D = \{d\}$. □

Except for $d = 1$ and $k \geq 3$ in the proof of Theorem 4, the construction there yields a connected k-partite graph and we have the following result.

Corollary 5 *Every singleton set of positive integers is a degree set of some connected k-partite graph, except $\{1\}$ for $k \geq 3$ in which case the k-partite graph is not connected.*

Now, we obtain the following result.

Theorem 6 *Every set of positive integers is a degree set of some connected k-partite graph, except $\{1\}$ for $k \geq 3$ in which case the k-partite graph is not connected.*

Proof. Let $d_1, d_2, \dots, d_n$ be positive integers. We will show that there is a connected k-partite graph $G(V_1,V_2,\dots,V_k)$ with degree set $D=\left\{d_1, \sum_{i=1}^{2} d_i, \dots, \sum_{i=1}^{n} d_i\right\}$, except when $d = 1$ and $k \geq 3$ in which case the k-partite graph is not connected.

The case $k = 2$ and $k = 3$ are respectively given in Theorem 2 and Theorem 3. Also the case $k = 1$ follows by Corollary 5. So, we assume $k \geq 4$ and $n \geq 2$.

For $k = 4$, construct a 4-partite graph $G(V_1, V_2, V_3, V_4)$ as follows. Let

$$\begin{aligned}
V_1 &= A_{11} \cup A_{12} \cup A_{13} \cup \dots \cup A_{1(n-1)} \cup A_{1n},\\
V_2 &= A_{21} \cup A_{22} \cup A_{23} \cup \dots \cup A_{2(n-1)},\\
V_3 &= A_{31} \cup A_{32} \cup A_{33} \cup \dots \cup A_{3(n-1)} \cup A_{3n},\\
V_4 &= A_{41} \cup A_{42} \cup A_{43} \cup \dots \cup A_{4(n-1)} \cup A_{4n},
\end{aligned}$$

with $A_{1p} \cap A_{1q} = \emptyset$, $A_{2p} \cap A_{2q} = \emptyset$, $A_{3p} \cap A_{3q} = \emptyset$,$A_{4p} \cap A_{4q} = \emptyset$ $(p \neq q)$, $|A_{1j}| = d_j$ for all j, $1 \leq j \leq n$, $|A_{2j}| = d_1 + d_2 + \dots + d_j$ for all j, $1 \leq j \leq n-1$, $|A_{3j}| = d_j$ for all j, $1 \leq j \leq n$, $|A_{41}| = d_2$, $|A_{4j}| = d_1 + d_2 + \dots + d_{j-1}$ for all j, $2 \leq j \leq n$. Let there be an edge **(i)** from each vertex of A_{1j} to every vertex of A_{3r} whenever $j \geq r$, **(ii)** from each vertex of A_{11} to every vertex of A_{41}, **(iii)** from each vertex of A_{2j} to every vertex of $A_{3(j+1)}$, and **(iv)** from each vertex of A_{2j} to every vertex of $A_{4(j+1)}$. Then, the degrees of the vertices of $G(V_1, V_2, V_3, V_4)$ are as follows.

$$d_{a_{11}} = |A_{31}| + |A_{41}| = d_1 + d_2, \text{ for all } a_{11} \in A_{11},$$

$$\text{for } 2 \leq j \leq n, \; d_{a_{1j}} = \sum_{r=1}^{j} |A_{3r}| = \sum_{r=1}^{j} d_r, \text{ for all } a_{1j} \in A_{1j},$$

$$\text{for } 1 \leq j \leq n-1, \; d_{a_{2j}} = |A_{3(j+1)}| + |A_{4(j+1)}| = d_{j+1} + d_1 + \cdots + d_j = \sum_{r=1}^{j+1} d_r, \text{ for all } a_{2j} \in A_{2j},$$

$$d_{a_{31}} = \sum_{r=1}^{n} |A_{1r}| = \sum_{r=1}^{n} d_r, \text{ for all } a_{31} \in A_{31},$$

$$\text{for } 2 \leq j \leq n, \; d_{a_{3j}} = \sum_{r=j}^{n} |A_{1r}| + |A_{2(j-1)}| = \sum_{r=j}^{n} d_r + d_1 + \cdots + d_{j-1} = \sum_{r=1}^{n} d_r, \text{ for all } a_{3j} \in A_{3j},$$

$$d_{a_{41}} = |A_{11}| = d_1, \text{ for all } a_{41} \in A_{41},$$

$$\text{for } 2 \leq j \leq n, \; d_{a_{4j}} = |A_{2(j-1)}| = d_1 + d_2 + \cdots + d_{j-1} = \sum_{r=1}^{j-1} d_r, \text{ for all } a_{4j} \in A_{4j}.$$

Therefore, degree set of $G(V_1, V_2, V_3, V_4)$ is $D = \{d_1, \sum_{i=1}^{2} d_i, \ldots, \sum_{i=1}^{n} d_i\}$.

If $k \geq 5$ is odd say $k = 2m + 3$ where $m \geq 1$, construct a $2m + 3$-partite graph $G(V_1, V_2, \ldots, V_{2m+3})$ as follows.

Let

$$\begin{aligned}
V_1 &= A_{11} \cup A_{12} \cup A_{13} \cup \cdots \cup A_{1(n-1)} \cup A_{1n}, \\
V_2 &= A_{21} \cup A_{22} \cup A_{23} \cup \cdots \cup A_{2(n-1)}, \\
V_3 &= A_{31} \cup A_{32} \cup A_{33} \cup \cdots \cup A_{3(n-1)} \cup A_{3n}, \\
V_4 &= A_{41} \cup B_{42} \cup A_{42} \cup A_{43} \cup \cdots \cup A_{4(n-1)} \cup A_{4n}, \\
V_5 &= A_{51} \cup B_{52}, \\
V_6 &= A_{61} \cup B_{62}, \\
&\vdots \\
V_{2m+2} &= A_{(2m+2)1} \cup B_{(2m+2)2}, \\
V_{2m+3} &= A_{(2m+3)1} \cup B_{(2m+3)2},
\end{aligned}$$

with $A_{1p} \cap A_{1q} = \emptyset$, $A_{2p} \cap A_{2q} = \emptyset$, $A_{3p} \cap A_{3q} = \emptyset$, $A_{4p} \cap A_{4q} = \emptyset$, $(p \neq q)$, $A_{4p} \cap B_{42} = \emptyset$, $A_{p1} \cap B_{p2} = \emptyset$, $|A_{1j}| = d_j$ for all j, $1 \leq j \leq n$, $|A_{2j}| = d_1 + d_2 + \cdots + d_j$ for all j, $1 \leq j \leq n-1$, $|A_{3j}| = d_j$ for all j, $1 \leq j \leq n$, $|A_{i1}| = d_2$, for all i, $4 \leq i \leq 2m+3$, $|B_{i2}| = d_1$ for all i, $4 \leq i \leq 2m+3$, $|A_{4j}| = d_1 + d_2 + \cdots + d_{j-1}$ for all $j, 2 \leq j \leq n$. Let there be an edge **(i)** from each vertex of A_{1j} to every vertex of A_{3r} whenever $j \geq r$, **(ii)** from each vertex of A_{11} to every vertex of A_{41}, **(iii)** from each vertex of A_{2j} to every vertex of $A_{3(j+1)}$, **(iv)** from each vertex of A_{2j} to every vertex of $A_{4(j+1)}$, **(v)** from each vertex of A_{i1} to every vertex of $A_{(i+1)1}$ for all even $i \geq 4$, **(vi)** from each vertex of B_{i2} to every vertex of $B_{(i+1)2}$ for all even $i \geq 4$, and **vii** from each vertex of B_{i2} to every vertex of $A_{(i+1)1}$ for all $i \geq 4$. Then the degrees of the vertices of $G(V_1, V_2, \ldots, V_{2m+3})$ are as follows.

$$d_{a_{11}} = |A_{31}| + |A_{41}| = d_1 + d_2, \text{ for all } a_{11} \in A_{11},$$

$$\text{for } 2 \leq j \leq n, \; d_{a_{1j}} = \sum_{r=1}^{j} |A_{3r}| = \sum_{r=1}^{j} d_r, \text{ for all } a_{1j} \in A_{1j},$$

$$\text{for } 1 \leq j \leq n-1, \; d_{a_{2j}} = |A_{3(j+1)}| + |A_{4(j+1)}| = d_{j+1} + d_1 + \cdots + d_j = \sum_{r=1}^{j+1} d_r, \text{ for all } a_{2j} \in A_{2j},$$

$$d_{a_{31}} = \sum_{r=1}^{n} |A_{1r}| = \sum_{r=1}^{n} d_r, \text{ for all } a_{31} \in A_{31},$$

$$\text{for } 2 \leq j \leq n, \; d_{a_{3j}} = \sum_{r=j}^{n} |A_{1r}| + |A_{2(j-1)}| = \sum_{r=j}^{n} d_r + d_1 + \cdots + d_{j-1} = \sum_{r=1}^{n} d_r, \text{ for all } a_{3j} \in A_{3j},$$

$$d_{a_{41}} = |A_{11}| + |A_{51}| = d_1 + d_2, \text{ for all } a_{41} \in A_{41},$$

$$\text{for } 2 \leq j \leq n, \; d_{a_{4j}} = |A_{2(j-1)}| = d_1 + d_2 + \cdots + d_{j-1} = \sum_{r=1}^{j-1} d_r, \text{ for all } a_{4j} \in A_{4j},$$

$$\text{for even } 4 \leq i \leq 2m+2, \; d_{b_{i2}} = |A_{(i+1)1}| + |B_{(i+1)2}| = d_2 + d_1 = d_1 + d_2, \text{ for all } b_{i2} \in B_{i2},$$

$$\text{for odd } 5 \leq i \leq 2m+1, \; d_{b_{i2}} = |B_{(i-1)2}| + |A_{(i+1)1}| = d_1 + d_2, \text{for all } b_{i2} \in B_{i2},$$

$$d_{b_{(2m+3)2}} = |B_{(2m+2)2}| = d_1, \text{ for all } b_{(2m+3)2} \in B_{(2m+3)2},$$

$$\text{for odd } 5 \leq i \leq 2m+3, \; d_{a_{i1}} = |A_{(i-1)2}| + |B_{(i-1)2}| = d_2 + d_1 = d_1 + d_2, \text{ for all } a_{i1} \in A_{i1},$$

for even $6 \le i \le 2m+2$, $d_{a_{i1}} = |A_{(i+1)1}|+|B_{(i-1)2}| = d_2+d_1 = d_1+d_2$,

for all $a_{i1} \in A_{i1}$.

Therefore, degree set of $G(V_1, V_2, \dots, V_{2m+3})$ is $D = \{d_1, \sum_{i=1}^{2} d_i, \dots, \sum_{i=1}^{n} d_i\}$.

Now, assume $k \ge 6$ is even, say $k = 2m + 4$ where $m \ge 1$. We add a new partition set V_{2m+4} to the above constructed $2m + 3$-partite graph $G(V_1, V_2, \dots, V_{2m+3})$ with $|V_{2m+4}| = d_2$ and let there be an edge from each vertex of V_{2m+4} to every vertex of $B_{(2m+3)2}$ so that we obtain a $2m+4$-partite graph $G(V_1,V_2,\dots,V_{2m+3},V_{2m+4})$. It is clear that in $G(V_1, V_2, \dots, V_{2m+3}, V_{2m+4})$ the degrees of all the vertices from the partite sets $V_1, V_2, \dots, V_{2m+3}$ remain unchanged except the vertices in $B_{(2m+3)2}$ (of V_{2m+3}) whose degrees are increased to $d_1 + d_2$ and the degree of each vertex in V_{2m+4} is d_1. Therefore, the degree set of $G(V_1, V_2, \dots, V_{2m+3}, V_{2m+4})$ is $D = \{d_1, \sum_{i=1}^{2} d_i, \dots, \sum_{i=1}^{n} d_i\}$.

We note that in above construction, all the $k-partite$ graphs are connected except when $d_1 = 1$ and $k \ge 3$. □

Finally, we have the following result.

Theorem 7 *Every set of non-negative integers is a degree set of some* k*-partite graph.*

Proof. Let $d_1, d_2, \dots, d_n$ be non-negative integers with $d_2, d_3, \dots, d_n > 0$. We will show there is a k-partite graph $G(V_1, V_2, \dots, V_k)$ with the degree set $D = \{d_1, \sum_{i=1}^{2} d_i, \dots, \sum_{i=1}^{n} d_i\}$.

First assume that $d_1 = 0$. For $n = 1$, consider a null k-partite graph $G(V_1, V_2, \dots, V_k)$ with $|V_i| = 1$ for all i, $1 \le i \le k$. Then for $1 \le i \le k$, $d_{v_i} = 0 = d_1$, for all $v_i \in V_i$. Therefore, degree set of $G(V_1, V_2, \dots, V_k)$ is $D = \{d_1\}$.

Now let $n > 1$. Since $d_2, d_3, \dots, d_n$ are positive integers, therefore by Theorem 6, there exists a k-partite graph $G(W_1, W_2, \dots, W_k)$ with degree set $D_1 = \{d_2, \sum_{i=2}^{3} d_i, \dots, \sum_{i=2}^{n} d_i\}$. We construct another k-partite graph $G(V_1, V_2, \dots, V_k)$ as follows.

Let $V_1 = W_1 \cup \{v\}, V_2 = W_2, \dots, V_k = W_k$. Then the degree of the vertex v is zero, that is, $d_v = 0 = d_1$ and the degrees of all the vertices of the partition set $W_1, W_2, \dots, W_k$ remain unchanged in $G(V_1, V_2, \dots, V_k)$. Therefore $G(V_1,V_2,\dots,V_k)$ is a k-partite graph with degree set $D = \{d_1, \sum_{i=1}^{2} d_i, \dots, \sum_{i=1}^{n} d_i\}$.

Now assume that $d_1 > 0$. Then $d_1, d_2, \ldots, d_n$ are the positive integers and therefore by Theorem 6, there exists a k-partite graph $G(V_1, V_2, \ldots, V_k)$ with degree set $D = \{d_1, \sum_{i=1}^{2} d_i, \ldots, \sum_{i=1}^{n} d_i\}$. □

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