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# Numerical Study of Fluid Flow Through a Confined Porous Square Cylinder

Salah GUERBAAI[1], Mouna TOUIKER[2], Kamel MEFTAH[3]and Abdeslam OMARA[4]

[1] Department of Mechanical Engineering, Université 20 Août 1955- Skikda, e-mail: guer_salah@yahoo.fr
[2] Department of Physics, Université 20 Août 1955- Skikda, e-mail: Touikerm21@yahoo.fr
[3] University of Biskra, Laboratoire de Génie Energétique et Matériaux (LGEM), Faculty of Science and Technology, Biskra 07000, Algeria, e-mail: meftah.kamel@gmail.com
[4] Department of Physics, Université des frères Mentouri Constantine 1, e-mail: abdeslam_omara@yahoo.fr

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**Abstract:** A numerical study is performed to analyze steady state forced convection fluid flow through a confined porous square cylinder. The Darcy-Brinkman-Forchheimer model is adopted for the porous region. The finite volume method and the iterative SIMPLE algorithm are used to solve the governing equations. The results obtained are presented for the streamlines, variation of Nusselt number and drag coefficient for the range of conditions as $5 \leq Re \leq 40$ and $10^{-2} \leq Da \leq 10^{-6}$.

**Keywords:** Forced convection, porous square cylinder, drag coefficient, Darcy number, finite volume method.

## 1. Introduction

The study of flows past or through a cylinder, triangular or square prism has received considerable attention in recent years because of their wide applications in engineering, for example, the cooling of electrical components, heat exchangers and cooling towers.

Breuer et al. [1] studied the confined flow of an incompressible fluid around a square cylinder with a blockage ratio of 1/8 using two different numerical techniques, namely Lattice-Boltzmann automata (LBA) and a finite volume method (FVM). Gupta et al. [2] have investigated the two dimensions forced convective heat transfer of a non-Newtonian fluid past a square cylinder with a blockage ratio of 1/8. The same study was carried out by Paliwal et al. [3], but for unconfined flow and with a blockage ratio of 1/15 and by Dhiman et al. [4] for an isolated unconfined square cylinder. The effect of Reynolds and Prandtl numbers on heat transfer across a square cylinder was tested by Dhiman et al. [5]. Sharma et al. [6] analyzed the effect of buoyancy forces on the upward flow of an incompressible fluid around a square cylinder

**Nomenclature**

| | |
|---|---|
| $b$ | Side length of the square cylinder, *m* |
| $C_{Df}$ | Viscous drag coefficient |
| $C_{DP}$ | Pressure drag coefficient |
| $C_D$ | Total drag coefficient |
| $d$ | Particle diameter, *m* |
| $Da$ | Darcy number, $\kappa/b^2$ |
| $H$ | Height of the computational domain, *m* |
| $k$ | Thermal conductivity, *W/m K* |
| $L$ | Length of the computational domain, *m* |
| $Nu$ | Nusselt number |
| $P$ | Dimensionless pressure |
| $Pr$ | Prandtl number |
| $Re$ | Reynolds number |
| $R_k$ | Ratio of thermal conductivity of porous and fluid layers, $k_{eff}/k_f$ |
| $T$ | Temperature, K |
| $U$, $V$ | Dimensionless velocity components |
| $X$, $Y$ | Dimensionless Cartesian coordinates |
| *Greek symbols*: | |
| $\mu$ | Dynanic viscosity, $kg\ m^{-1}s^{-1}$ |
| $\Lambda$ | Viscosity ratio, $\mu_{eff}/\mu_f$ |
| $\varepsilon$ | Porosity |
| $\theta$ | Dimensionless temperature |
| $\varphi$ | Blockage ratio, $b$/H |
| $\kappa$ | Permeability of the material, $m^2$ |
| *Subscripts:* | |
| *eff* | Effective |
| *f* | Fluid |
| *fr* | Front face |
| *r* | Rear face |
| *t* | Top face |
| *b* | Bottom face |

Flow and heat transfer through permeable bodies has been a subject of great interest in these last years since they have a large contact surface and therefore tend to improve heat exchange compared to their solid counterparts. Alazmi and Vafai [7] analyzed in detail the different types of interfacial conditions between a porous medium and a fluid layer. The effect of Reynolds and Darcy numbers on the structure of a flow through a porous square cylinder was investigated by Yu et al. [8]. Wu and Wang [9] analyzed numerically an unsteady flow and convection heat transfer for a heated square porous cylinder in a channel. The heat transfer through an isolated porous square cylinder maintained at a constant temperature was examined by Dhinakaran and Ponmozhi [10], their results showed that the flow pattern through and around the porous cylinder depends much on the Darcy number of the porous medium. Mahdhaoui et al. [12] examined the effects of Darcy and Reynolds numbers on the flow structure and heat transfer through a porous square cylinder with a blockage ratio of 1/11. Their results show principally that for a low Darcy number, there is a recirculation zone after the porous medium, at Da=$10^{-6}$ the flow structure is similar to that of a solid square cylinder.

The aim of this work is to study the effects of Reynolds and Darcy numbers of the flow pattern through and around a porous cylinder placed between two parallel plates with a blockage ratio of 1/8.

## 2. Mathematical formulation

The geometric model studied is identical to that examined by Dhiman et al. [11] but in the case of a porous square cylinder which is exposed to a parabolic stream velocity as shown in *Fig.1*. The values of *Xu, Xd* and *H* are respectively *8.5b, 16.5b* and *8b.*

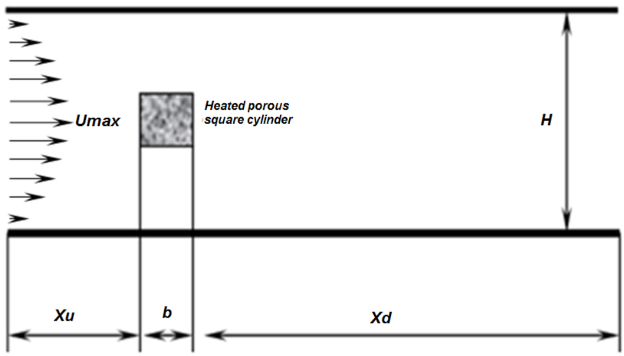


*Figure 1*: Schematic diagram of the physical system.

In the Cartesian coordinate system, the fundamental dimensionless governing equations based on Darcy-Brinkman-Forchheimer extended model are as follows:

Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

Momentum equations:

$$\frac{1}{\varepsilon}\left(U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\varepsilon\frac{\partial P}{\partial X} + \frac{\Lambda}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - -C\left(\varepsilon\frac{1}{ReDa}U + \frac{1.75}{\sqrt{150}}\frac{1}{\sqrt{Da}} \times \frac{\sqrt{U^2+V^2}}{\sqrt{\varepsilon}}U\right) \tag{2}$$

$$\frac{1}{\varepsilon}\left(U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\varepsilon\frac{\partial P}{\partial Y} + \frac{\Lambda}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) -$$

$$-C\left(\varepsilon\frac{1}{ReDa}V+\frac{1.75}{\sqrt{150}}\frac{1}{\sqrt{Da}}\times\frac{\sqrt{U^2+V^2}}{\sqrt{\varepsilon}}V\right) \quad (3)$$

Energy equation:

$$\frac{1}{\varepsilon}\left(U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}\right)=\frac{R_k}{RePr}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \quad (4)$$

$$\mathrm{C}=\begin{cases}0. & \text{outside the porous medium}\\ 1. & \text{in the porous medium}\end{cases} \quad (5)$$

$$\varepsilon=\begin{matrix}1. & \text{outside the porous medium}\\ 0<\epsilon<1 & \text{in the porous medium}\end{matrix} \quad (6)$$

The dimensionless variables introduced are given as:

$$X=\frac{x}{b};\ Y=\frac{y}{b};\ U=\frac{u}{u_{max}}\ ;\ V=\frac{v}{u_{max}}\ ;\ P=\frac{p}{\rho_f u_0^2}\ ;\ \theta=\frac{T-T_0}{\Delta T} \quad (7)$$

The porosity and the Darcy number are related by the Carman-Kozeny equation given by:

$$Da=\frac{1}{180.b^2}\frac{\varepsilon^3 d^2}{(1-\varepsilon)^2}=\frac{\kappa}{\mathrm{b}^2}\,. \quad (8)$$

*d* is the mean particle diameter in the porous cylinder, each particle may be of 100 μm in diameter [10].

The dimensionless boundary conditions are:

$$\begin{matrix}U=0.\ ;\ V=0.\ ;\ \frac{\partial\theta}{\partial Y}=0 & \text{at upper and lower boundary}\\ U=0.5Y(1-0.125Y)\ ;\ V=0\ ;\theta=0 & \text{at the inlet boundary}\\ \theta=1 & \text{on the surface of the square cylinder}\\ \frac{\partial U}{\partial X}=\frac{\partial V}{\partial X}=\frac{\partial\theta}{\partial X}=0 & \text{at the exit boundary}\\ \mu_f\frac{\partial u_n}{\partial n}\Big|_f=\mu_{eff}\frac{\partial u_n}{\partial n}\Big|_p & \text{at the porous} -\text{ fluid interface}\end{matrix} \quad (9)$$

where *n* represents the direction normal to the surface of the cylinder and

$$\mu_{eff}=\frac{\mu_f}{\varepsilon} \quad (10)$$

The heat transfer from the porous cylinder to the fluid is evaluated from the local Nusselt number calculated as:

$$Nu = -\frac{k_{eff}}{k_f}\frac{\partial \theta}{\partial n}. \quad (11)$$

$R_k = \frac{k_{eff}}{k_f}$ is taken equal to 1.

The total drag coefficient $C_D$ on the cylinder is given by:

$$C_D = C_{D_P} + C_{D_f} \quad (12)$$

The drag coefficients due to the pressure and viscous forces can be obtained by evaluating the following integrals:

$$C_{D_P} = 2.0\int_0^1 \left(P_{fr} - P_r\right) dY,\; C_{D_f} = \frac{2.0}{Re}\int_0^1 \left[\left(\frac{\partial U}{\partial Y}\right)_b + \left(\frac{\partial U}{\partial Y}\right)_t\right] dX \quad (13)$$

## 3. Solution method and validation

### *3.1. Solution method*

A control volume approach presented by Patankar [13] is used to discretize the dimensionless governing equations (1) - (4). The semi-implicit method for pressure-linked equation (SIMPLE) algorithm was used for solving the coupled momentum and continuity equations using the corresponding boundary conditions given in equations (9). The convergence of the sequential iterative solution is obtained when:

$$max\left(\left(\emptyset_{i,j}^{k+1} - \emptyset_{i,j}^{k}\right)/\emptyset_{i,j}^{k+1}\right) \leq 10^{-5}, \quad (14)$$

where $\emptyset_{i,j}^{k}$ is the general dependent variable which can stand for *U*, *V*, or $\theta$ at every (*X*, *Y*) position of the discretized domain, *k* is the iteration step.

### *3.2. Code validation*

The currently used FORTRAN code is firstly validated in terms of the drag coefficient and the recirculation length with the published results of Dhiman et al. [11], Breuer et al. [1] and Gupta et al. [2] for the case of a flow of an incompressible fluid (Pr = 0.7) across a solid square cylinder with a blockage ratio of 1/8 as can be shown in *Table 1*.

*Table 1*: Comparison of drag coefficient ($C_D$ )and recirculation length ($L_r$) with the literature for the case of solid square cylinder

| | *Re = 20* | | *Re = 30* | | *Re = 40* | |
|---|---|---|---|---|---|---|
| | $L_r$ | $C_D$ | $L_r$ | $C_D$ | $L_r$ | $C_D$ |
| Present work | 1.05 | 2.44 | 1.63 | 2.00 | 2.17 | 1.76 |
| Dhiman et al. [11] | 1.05 | 2.44 | 1.62 | 1.99 | 2.17 | 1.75 |
| Breuer et al. [1] | 1.04 | 2.50 | 1.60 | 2.00 | 2.15 | 1.70 |
| Gupta et al. [2] | 0.90 | 2.45 | 1.40 | 2.06 | 1.90 | 1.86 |

A second validation was made with a comparison of the streamlines drawn by Mahdhaoui et al. [12] for a laminar flow through a porous square cylinder with blockage ratio of 1/11 and for different values of Darcy number (Da = $10^{-2}$, $10^{-4}$ and $10^{-6}$) at Re = 20. This comparison is presented in *Fig.2*.

The results show that the present study is in good agreement with the published results.

## 4. Results and discussion

A numerical study was conducted to investigate the effects of porous proprieties on several hydrodynamic and thermal parameters such as wake structure, streamlines, drag coefficient and heat transfer rate. The results have been obtained for the following range of parameters:

Reynolds number: 1 < Re < 40, Darcy number: Da = $10^{-6}$, $10^{-4}$, $10^{-3}$, $10^{-2}$ and the corresponding porosity: $\varepsilon$ = 0.629, 0.8, 0.977, 0.993 respectively.

*Fig.3 s*hows the effects of Reynolds and Darcy numbers on the flow structure. Streamlines near the square cylinder are given for Re = 10, 20 and 40 and *Da* = $10^{-2}$, $10^{-3}$, $10^{-4}$ and $10^{-6}$. For high values of Darcy number, the fluid encounters a low resistance as it passes through the porous medium with the complete absence of recirculation cells after the obstacle. With the decrease in permeability of the obstacle, two counter rotating eddies are formed behind the square cylinder, whose size increases with increasing of the Reynolds number.

For very small Darcy number values (*Da* = $10^{-6}$) the flow structure is similar to that of the flow through a solid square cylinder and the fluid cannot pass through the square cylinder.

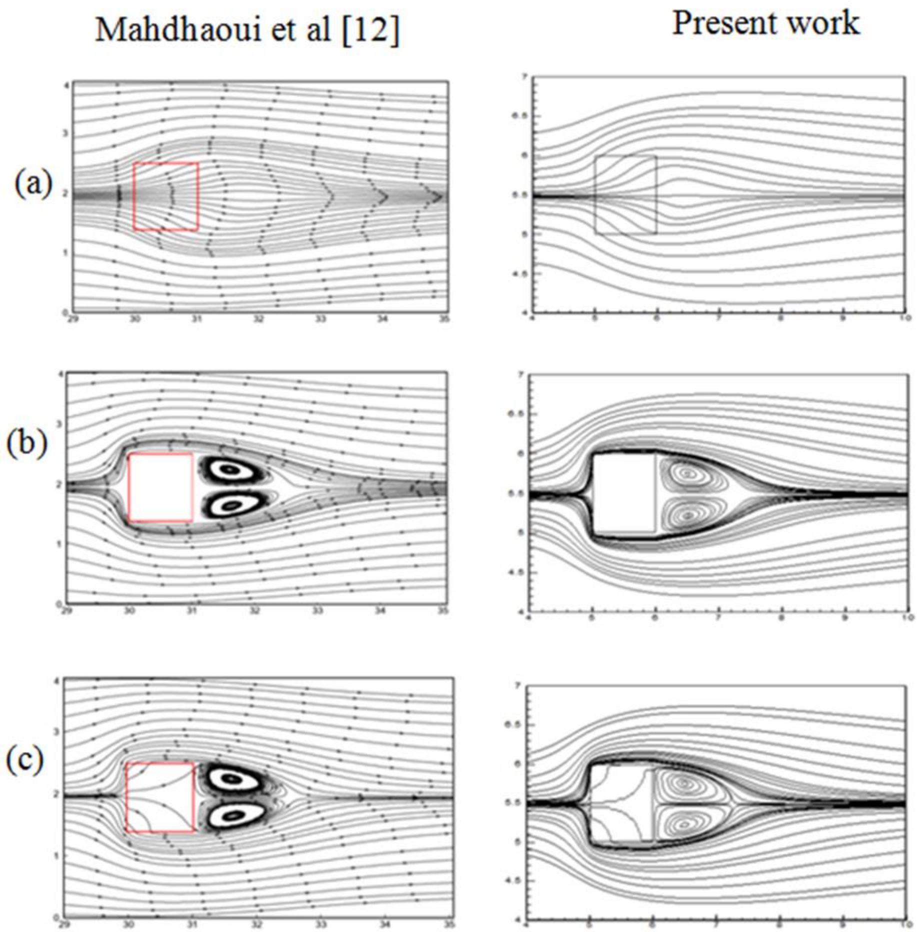


*Figure 2*: Comparison of stream lines between the present numerical solution and those of Mahdhdaoui et al. [12] at *Re* = 20 and at differents *Da*. (a) $Da = 10^{-2}$, (b) $Da = 10^{-4}$ and (c) $Da = 10^{-6}$ .

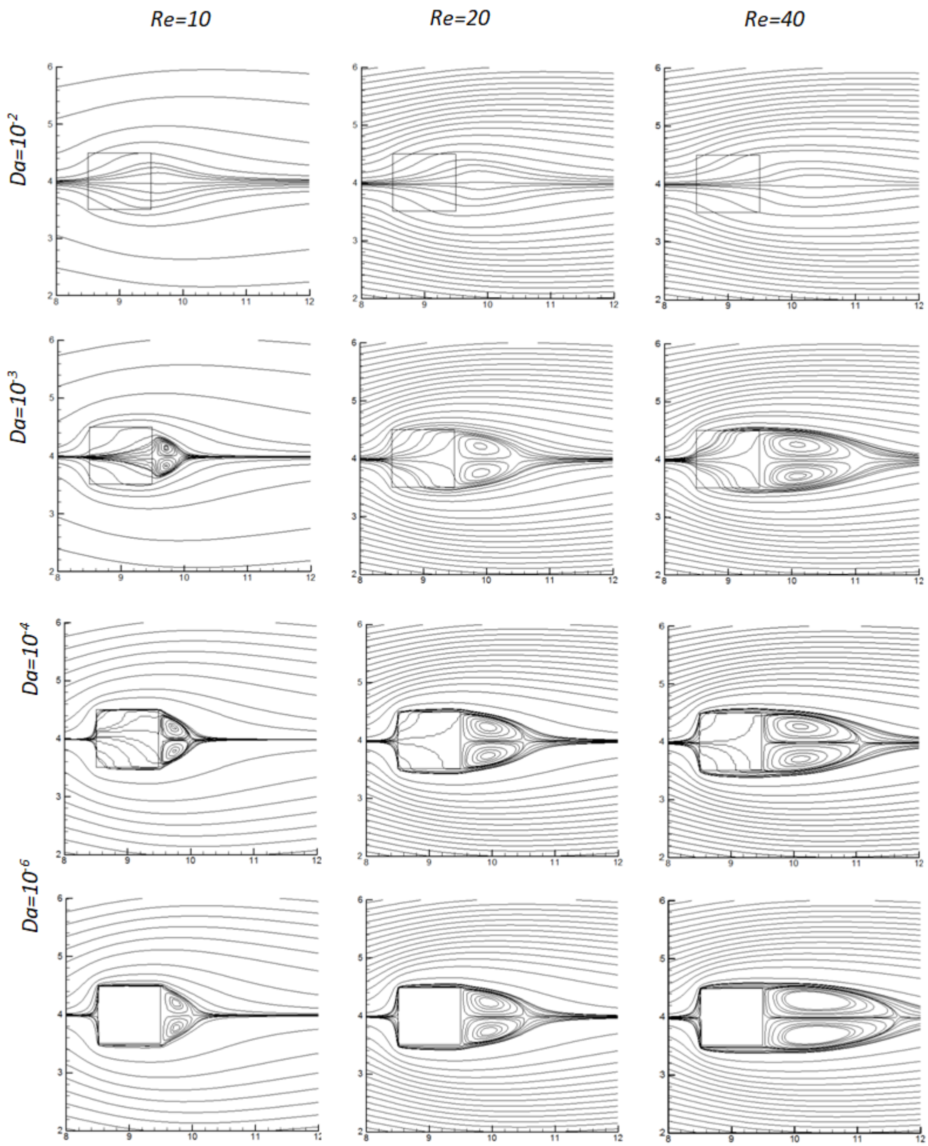


*Figure 3*: Streamline plots for various Reynolds and Darcy numbers.

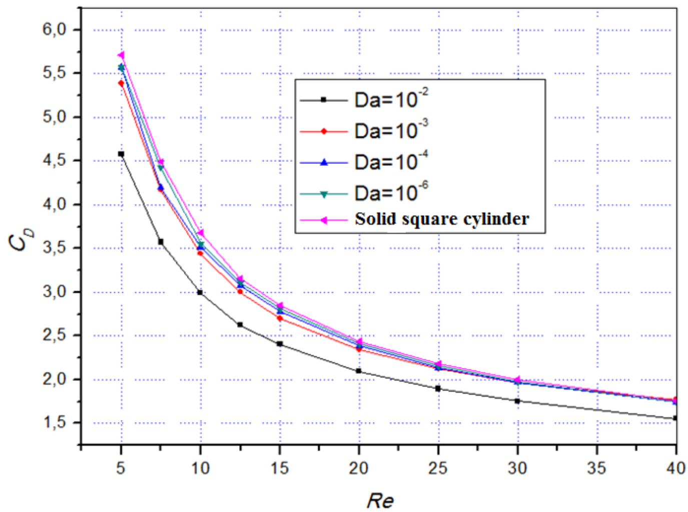


*Figure 4*: Variation of the drag coefficient with Reynolds number for different Darcy numbers.

The value of the drag coefficient decreases with the increase in the value of the Reynolds number which results in a depletion of the viscous layer.

The high values of the permeability of the material cause a low resistance to the flow of the fluid and thus a reduction of the pressure gradient inside the porous medium, which causes a decrease in the value of the drag coefficient as shown in *Fig 4.*

*Fig. 5* shows the variation of the average Nusselt number for different Darcy and Reynolds numbers. It can clearly be seen that the heat transfer rate increases with the increase in Reynolds number on the one hand and with the increase in the permeability of the material of the obstacle on the other hand.

The variation of the length of the recirculation region with Reynolds number is shown in *Fig. 6*. It shows that the wake length decreases with the increasing Darcy number for Re < 35 and increases for Re > 35. This same observation was reported by Dhinakaran and Ponmozhi [10].

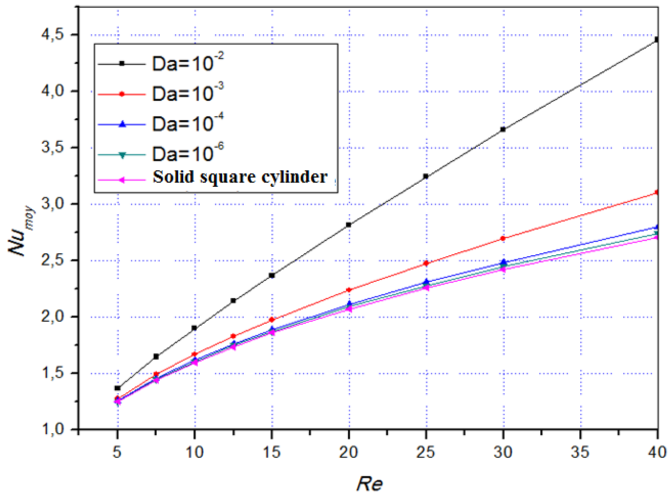


*Figure 5*: Variation of the average Nusselt number with Reynolds number for different Darcy numbers.

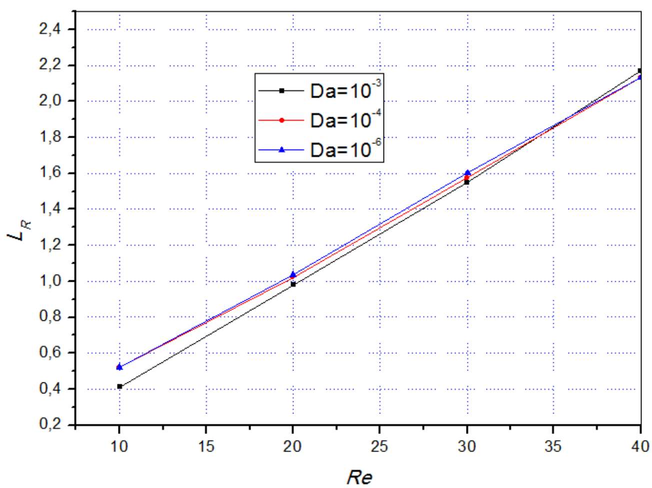


*Figure 6*: Variation of recirculation wake length with Reynolds number for different Darcy numbers.

## 5. Conclusion

The laminar flow of an incompressible fluid around and through a square cylinder maintained at a constant temperature and placed between two parallel plates has been studied. The equations governing this flow are solved numerically by the finite volume method using the SIMPLE algorithm. The effects of Reynolds and Darcy numbers on the flow and heat transfer characteristics have been presented. The following conclusions have been drawn from the studies presented:

The drag coefficient increases with the decrease in the Darcy number and this approximates the case of a flow through a solid body for Da = $10^{-6}$.

For high permeability, the fluid receives a slight resistance to penetration into the square cylinder and thus the absence of the recirculation zone for different values of the Reynolds number.

The length of the recirculation zone increases with the increase of the number of Reynolds and decreases slightly with the increase of the Darcy number for Re < 35.

The transfer rate increases with the increase of the Reynolds number on the one hand and with the increase of the permeability of the material on the other hand.

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