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Lyapunov-Based Frequency-Shift Power Control of Induction-Heating Converters with Hybrid Resonant Load

András KELEMEN[1], Nimród KUTASI[2]

[1, 2] Department of Electrical Engineering, Faculty of Technical and Human Sciences,
Sapientia University, Tîrgu Mureş, Romania,
e-mail: kandras@ms.sapientia.ro, kutasi@ms.sapientia.ro

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Abstract: The frequency-shift method is an attractive choice for power control of induction-heating inverters due to the simplicity of the power circuit. However, frequency-shift control proves to be a challenging task in case of practical resonant loads with high quality factor and uncertain circuit parameters. The paper presents a bilinear large-signal model of the induction-heating inverter with hybrid LLC resonant load. A control law is proposed, which is based on the Lyapunov stability theory. Moreover, an adaptive control method is presented to handle the uncertainty concerning the nominal values of the state variables. The theoretical results are illustrated by numerical simulation.

Keywords: Induction heating, resonant inverter, energy in the increment, Lyapunov stability, d-q model.

1. Introduction

The technological task is heating metals by means of eddy currents induced directly in the work-piece by a strong and variable magnetic field produced by a high-intensity alternative current flowing through an inductor. The inductor is a component of a resonant circuit fed by a power electronic load-resonant inverter. The inductor is designed to fulfill the main technological requirement of heating the work-piece. For this purpose a magnetic field is necessary with a certain distribution in space and evolution in time, able to produce by eddy current losses the required heat pattern.

The power electronic converter has to deliver power at frequencies that are close to the resonance frequency of the load. The task is finding power control methods able to create low loss switching conditions of the power semiconductor devices, while keeping reduced complexity of the heating equipment [1],[2].

This paper proposes a power control method of load-resonant inverters by means of the operation frequency, named “frequency-shift control”, based on the Lyapunov stability theory.

The paper is organized as follows. Section 2 presents challenges of frequency-shift control of induction-heating voltage inverters. The bilinear large-signal low-frequency d-q model of the high-frequency inverter with hybrid LLC resonant load is introduced in Section 3. Construction of Lyapunov-based control laws for the above system is presented in Section 4 in case of known circuit parameters and known steady-state (nominal) values of the state variables. Section 5 introduces an estimation method for handling uncertain nominal values of the state variables. The proposed methods are verified by MatLab Simulink simulation. Finally, concluding remarks are presented in Section 7.

2. Considerations on frequency-shift power control

Schematic of the induction-heating converter with voltage-fed load-resonant inverter is shown in *Fig. 1.a*. The frequency-shift power control method is based on the frequency characteristics of the resonant load (*Fig. 1.b* presents the frequency characteristics of a particular load). Consequently the control characteristics are very much influenced by the load parameters and the power control range is relatively reduced. Some of these parameters, like L_s and C_p are known accurately and are subject only to small variations during the heating process. The L_{is} inductance of the heating inductor is generally known with less accuracy, and is subject to relatively small changes during the heating process. These changes are reflected in relatively small variations of the resonance frequency. However, large variation of power may result especially in case of high quality factor. The most uncertain load parameter, which is also most influenced during the heating process, is the R_{is} equivalent resistance of the inductor with work-piece. This parameter may change by an order of magnitude due to large variations with temperature of the resistivity and permeability, which are anyway known with low accuracy. In the same time, permeability is much dependent on the intensity of the magnetic field.

A classical frequency-shift control structure is shown in *Fig. 2*.

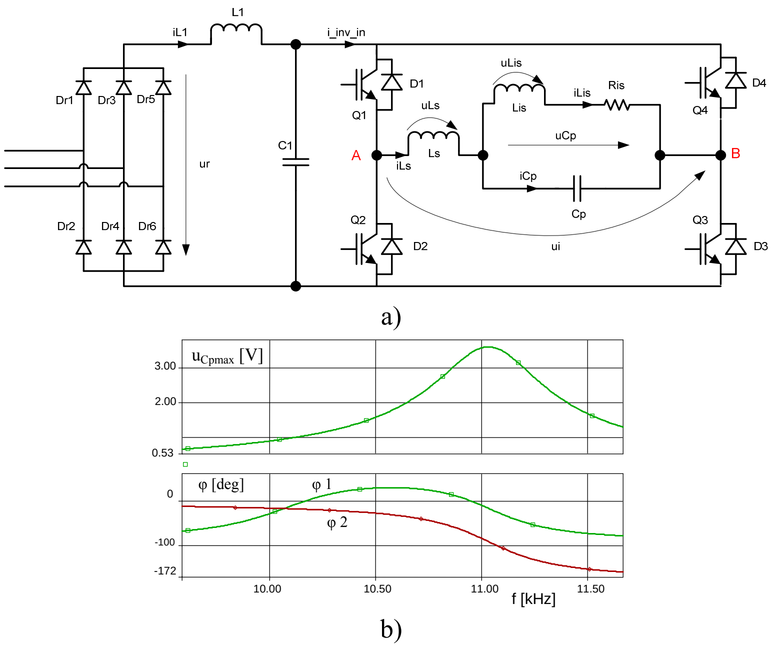


Figure 1: Schematic of the induction-heating converter (a) and frequency characteristics of the LLC hybrid resonant load (b). $u_{Cp\,\max}$ is the amplitude of the capacitor tank voltage, φ_1 denotes the phase shift between the inverter output current and voltage, while φ_2 denotes the phase shift between the capacitor tank voltage and inverter output voltage.

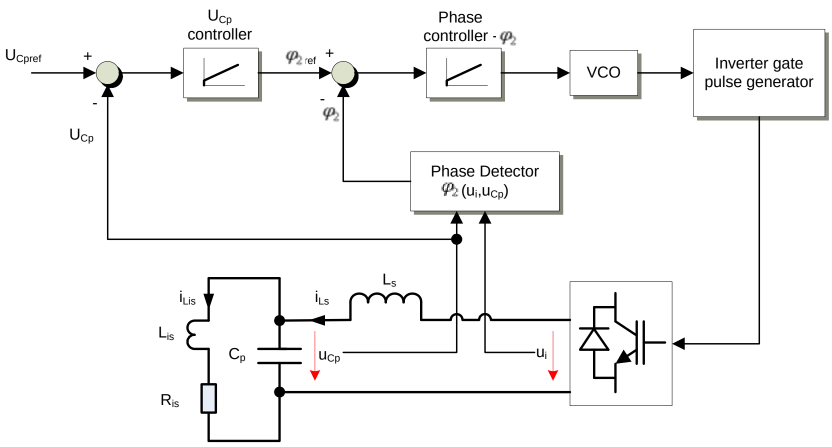


Figure 2: Traditional cascade-type frequency-shift control structure.

3. The model of the load-resonant inverter

For investigation of power control by means of frequency, we need converter models that are simple enough to be included in dynamic models of frequency and power control structures. Modeling of power electronic converters generally needs a hierarchic approach and development of modeling techniques tailored to the specific problem to be solved. In our approach rules are established that assure certain desired (soft) switching conditions by proper choice of the switching instant. Only those power control methods are accepted, that follow these rules. From this moment, control design, i.e. analysis of long-term closed-loop operation is made caring neither about the switching process, nor about the instantaneous values of the circuit variables.

In case of the frequency-shift control, the resonant load is fed by the inverter with full square-wave output voltage, thus the switching instant is strongly coupled with the switching frequency. The aim is the derivation of a model for the resonant inverter, which allows the analysis of the envelope of circuit variables (only variation of magnitudes is of concern). The model is built for continuous current mode operation, valid in case of frequency-shift control. The inverter output voltage is represented by its fundamental component.

The low-frequency model (*Fig. 4b* and *Fig. 5*) is obtained by eliminating the high-frequency terms from the equations written for the complex high-frequency circuit (*Fig. 4a*), which is derived from the original and the orthogonal circuit (*Fig. 3*) [3].

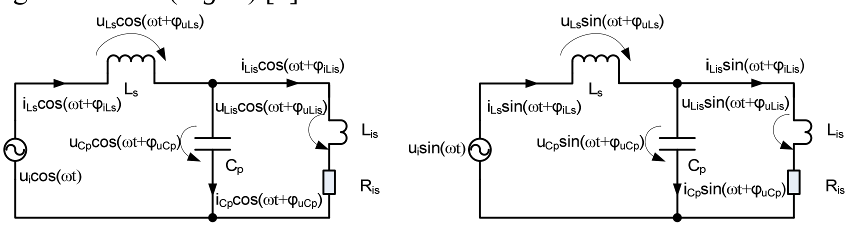
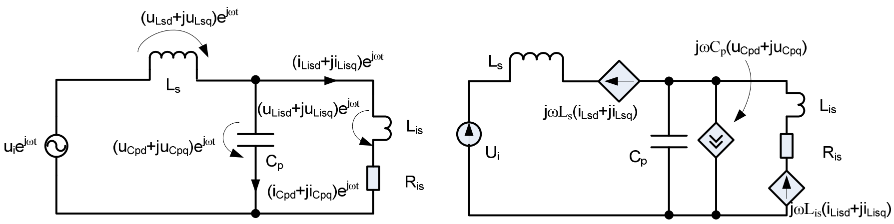


Figure 3: The inverter with resonant load and its orthogonal circuit.



a.) b.)

Figure 4: The complex representation (a) and the low-frequency complex d-q model of the resonant inverter (b).

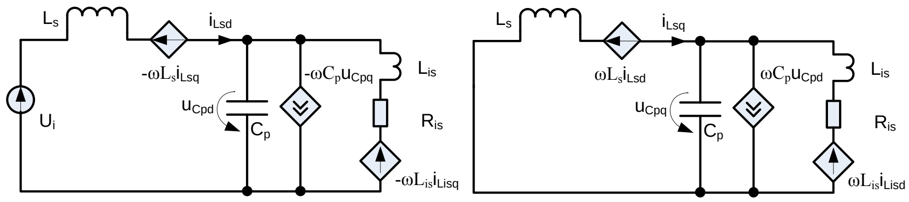


Figure 5: Low-frequency d and q circuits.

4. The Lyapunov-based frequency-shift power control

Using the non-linear model introduced in the previous section, we present an approach for development of the frequency-shift control law based on the Lyapunov stability theory.

The inverter has the structure from *Fig. 1.a*, with circuit parameters:

L_s= 20μH, L_{is}= 3.95μH, R_{is}= 0.03Ω, C_p= 63μF.

The amplitude of the rectangular inverter output voltage is assumed to be constant, with the peak value of its fundamental: u_{i1max}= 210V (lower-case notation has been used on purpose, to indicate the time-variable character of the amplitude).

The frequency characteristics of the inverter output current and load capacitor voltage are shown in *Fig. 1.b*.

Denoting by $\tilde{x}$ the -not necessarily small- deviation (increment) of a state variable x_T from its steady-state value x_n, it results:

$$[x_T] = [x_n] + [\tilde{x}] \tag{1}$$

In a similar way, in case of the angular frequency it results:

$$\omega = \omega_n + \tilde{\omega} \tag{2}$$

The state space equation system of the increments has the bilinear matrix form

$$[\dot{\tilde{x}}] = [A][\tilde{x}] + ([B][\tilde{x}] + [b])\tilde{\omega}\,, \tag{3}$$

with matrices detailed in (6).

A possible and advantageous choice of Lyapunov function candidate is the "energy in the increment" [4], [5]:

$$V(\tilde{x}) = \frac{1}{2}[\tilde{x}]^T[Q][\tilde{x}]\,, \tag{4}$$

with the positive defined diagonal matrix:

$$[Q] = diag([L_s \; L_s \; C_p \; C_p \; L_{is} \; L_{is}]). \tag{5}$$

$$[A] = \begin{bmatrix} 0 & \omega_n & -\frac{1}{L_s} & 0 & 0 & 0 \\ -\omega_n & 0 & 0 & -\frac{1}{L_s} & 0 & 0 \\ \frac{1}{C_p} & 0 & 0 & \omega_n & -\frac{1}{C_p} & 0 \\ 0 & \frac{1}{C_p} & -\omega_n & 0 & 0 & -\frac{1}{C_p} \\ 0 & 0 & \frac{1}{L_{is}} & 0 & -\frac{R_{is}}{L_{is}} & \omega_n \\ 0 & 0 & 0 & \frac{1}{L_{is}} & -\omega_n & -\frac{R_{is}}{L_{is}} \end{bmatrix}, \quad [B] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$[\tilde{x}] = [\tilde{i}_{Lsd} \; \tilde{i}_{Lsq} \; \tilde{u}_{Cpd} \; \tilde{u}_{Cpq} \; \tilde{i}_{Lisd} \; \tilde{i}_{Lisq}]^T, \tag{6}$$
$$[b] = [I_{Lsqn} \; -I_{Lsdn} \; U_{Cpqn} \; -U_{Cpdn} \; I_{Lisqn} \; -I_{Lisdn}]^T$$

Generally, for a positive scalar α, global stability of the bilinear system (3) is guaranteed by the control law:

$$\tilde{\omega} = -\alpha([B][\tilde{x}] + [b])^T [Q_1][\tilde{x}], \tag{7}$$

which assures a negative derivative of the function (4) along the system trajectories [5]. A short proof of this control law is given in [6], based on

$$\frac{d}{dt}(V([\tilde{x}])) = \frac{1}{2}[\tilde{x}]^T([A]^T[Q_1] + [Q_1][A])[\tilde{x}] + \frac{1}{2}\tilde{\omega}[\tilde{x}]^T([B]^T[Q_1] + [Q_1][B])[\tilde{x}] + \frac{1}{2}\tilde{\omega}([b]^T[Q_1][\tilde{x}] + [\tilde{x}]^T[Q_1][b]). \tag{8}$$

The existence of a symmetric, positive defined Q_1 that makes negative the first term of the derivative is guaranteed according to the Lyapunov equation:

$$[A]^T[Q_1] + [Q_1][A] = -[P][P]^T, \tag{9}$$

where $\{[P]^T, [A]\}$ is an observable pair.

The sum of the other two terms, equal to

$$N = \tilde{\omega}([\tilde{x}]^T[B]^T[Q_1][\tilde{x}] + [b]^T[Q_1][\tilde{x}]) \tag{10}$$

can be made negative by choosing the control law:

$$\tilde{\omega} = -\alpha([\tilde{x}]^T[B]^T[Q_1][\tilde{x}] + [b]^T[Q_1][\tilde{x}]) = -\alpha([B][\tilde{x}] + [b])^T[Q_1][\tilde{x}]. \tag{11}$$

The equation (9) is satisfied choosing $[Q_1] = [Q]$ (12).

$$[A]^T[Q] + [Q][A] = -R_{is}\,diag[0\;0\;0\;0\;1\;1] \leq 0, \tag{12}$$

a natural result taking into account that the resonant load is dissipative.

By direct calculation,

$$[B]^T[Q]+[Q][B]=[0] \tag{13}$$

and the control law takes the form

$$\tilde{\omega}=-\alpha[b]^T[Q][\tilde{x}]. \tag{14}$$

In order to assure monotonic control characteristics, the operation frequency is lower-limited by $\omega_{\inf}$, larger than the resonance frequency, and the control law becomes

$$\tilde{\omega}=\begin{cases}-\alpha[b]^T[Q][\tilde{x}] & for \quad -\alpha[b]^T[Q][\tilde{x}]>\omega_{\inf}-\omega_n \\ \omega_{\inf}-\omega_n & for \quad -\alpha[b]^T[Q][\tilde{x}]\le\omega_{\inf}-\omega_n\end{cases}. \tag{15}$$

Indeed, $\omega=\omega_n+\tilde{\omega}\ge\omega_{\inf}$ implies that $\tilde{\omega}\ge\omega_{\inf}-\omega_n$.

V(x) remains a Lyapunov function even in the saturation range of (15) because for positive values of α it results

$$-\alpha[b]^T[Q][\tilde{x}]\le\omega_{\inf}-\omega_n \quad\Rightarrow\quad [b]^T[Q][\tilde{x}]\ge\frac{\omega_n-\omega_{\inf}}{\alpha}>0, \tag{16}$$

and the third term of (8) is

$$\tilde{\omega}\left([b]^T[Q][\tilde{x}]\right)\le(\omega_{\inf}-\omega_n)\frac{\omega_n-\omega_{\inf}}{\alpha}\le 0, \tag{17}$$

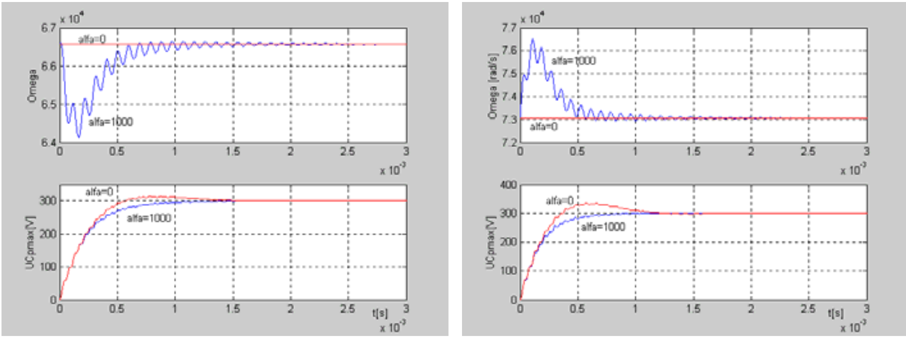
while $[b]^T[Q][\tilde{x}]=0$ implies $\tilde{\omega}=0$, and $\tilde{x}=0$.

Substituting the terms from (6) into (14), it results:

$$\tilde{\omega}=-\alpha\left(L_s I_{Lsqn}\tilde{i}_{Lsd}-L_s I_{Lsdn}\tilde{i}_{Lsq}+C_p U_{Cpqn}\tilde{u}_{Cpd}-C_p U_{Cpdn}\tilde{u}_{Cpq}+L_{is} I_{Lisqn}\tilde{i}_{Lisd}-L_{is} I_{Lisdn}\tilde{i}_{Lisq}\right) \tag{18}$$

The control law has been implemented in Matlab Simulink. *Fig. 6* shows the open-loop response of the LLC load to a step variation of the inverter output voltage, along with closed-loop evolution. *Fig. 6* shows the evolution of the u_{Cpmax} voltage towards a frequency which is lower (*Fig. 6.a*), respectively larger (*Fig. 6.*b) than the resonance frequency. It should be mentioned that knowledge of the steady-state values of the variables has been assumed. In case of the inverter shown in *Fig. 1* these are known from steady-state simulation results:
I_{Lsdn} = 182.2 A, I_{Lsqn} = -26.4 A, U_{Cpdn} = 176.3 V, U_{Cpqn} = -242.7 V,
I_{Lisdn} = -835.6 A, I_{Lisqn} = -765.9 A, ω_n = 66571 rad/sec, u_{i1maxn}=211.5V.

However, the main drawback of the above control method is the poor knowledge of the steady-state values, which explicitly appear in the control law (18). One reason of this uncertainty is the variation of the inductor parameters during the heating process.



a) b)

Figure 6: Comparison of the inverter's step responses for different gains α of the control law (18). α=0 means open-loop operation, i.e. response of the load to an "a.c. voltage step". In case (a) this means u_{i1max}=211V amplitude, ω_n=66750 rad/s angular frequency of the inverter output voltage fundamental, while the resonance occures at ω_0=69138 rad/s. The time diagrams show the evolution of the angular frequency (upper) and u_{Cpmax} load capacitor voltage amplitude (lower). In case (b), u_{i1max}=266 V, ω_n=73062 rad/s.

The next section presents a modified control method of the resonant capacitor tank voltage, able to estimate these values.

5. Estimation of the steady-state values

In order to handle the uncertainties of the steady-state values, the control law is modified in a manner that it is based on the estimated steady-state values and assures both global stability of the system and convergence of the estimation error towards zero. The Lyapunov candidate is modified (19)

$$V(\tilde{x}) = \frac{1}{2}[\tilde{x}]^T [Q][\tilde{x}] + \frac{1}{2}[\delta x_n]^T [K][\delta x_n], \tag{19}$$

with $[K]$ symmetric and positive defined, in order to include the estimation error vector

$$[\delta x_n] = [\tilde{x}_n] - [x_n]. \tag{20}$$

The derivative of the Lyapunov candidate becomes

$$\begin{aligned} \frac{dV(\tilde{x})}{dt} &= \frac{1}{2}[\tilde{x}]^T \left([A]^T [Q] + [Q][A]\right)[\tilde{x}] + \frac{1}{2}[\tilde{x}]^T \left([B]^T [Q] + [Q][B]\right)[\tilde{x}]\tilde{\omega} \\ &+ \frac{1}{2}\left([b]^T [Q][\tilde{x}] + [\tilde{x}]^T [Q][b]\right)\tilde{\omega} + \frac{1}{2}[\delta \dot{x}_n]^T [K][\delta x_n] + \frac{1}{2}[\delta x_n]^T [K][\delta \dot{x}_n] \end{aligned} \tag{21}$$

The sum of the last two terms from (21) can be brought to the form:

$$L=\frac{1}{2}[\delta\dot{x}_n]^T[K][\delta x_n]+\frac{1}{2}[\delta x_n]^T[K][\delta\dot{x}_n]=L_1\tilde{\omega}\ \text{ if} \tag{22}$$

$$[\delta\dot{x}_n]=\tilde{\omega}[F]. \tag{23}$$

It results [6]:

$$L=\frac{1}{2}\tilde{\omega}\left([F]^T[K][\delta x_n]+[\delta x_n]^T[K][F]\right). \tag{24}$$

The sum of the last five terms from (21) is made negative by the choice:

$$\tilde{\omega}=-\alpha\left([\tilde{x}]^T[B]^T[Q][\tilde{x}]+[b]^T[Q][\tilde{x}]+L_1\right)=-\alpha\left(([B][\tilde{x}]+[b])^T[Q][\tilde{x}]+L_1\right) \tag{25}$$

Generally the quantity $\left(([B][\tilde{x}]+[b])^T[Q]\right)^T=[Q]([B][\tilde{x}]+[b])$ can be measured easily [5] and $([\tilde{x}]-[\delta x_n])=[x_T]-[x_n]-[\delta x_n]=[x_T]-([x_n]+[\delta x_n])=[x_T]-[\tilde{x}_n]$ is known, because $[x_T]$ is the measured state vector and $[\tilde{x}_n]$ is the estimate of the steady state. Consequently, it would be useful if (25) was brought to the form

$$\tilde{\omega}=-\alpha([B][\tilde{x}]+[b])^T[Q]([\tilde{x}]-[\delta x_n]),\ \text{choosing} \tag{26}$$

$$L_1=-([B][\tilde{x}]+[b])^T[Q][\delta x_n] \tag{27}$$

Thus, we need

$$L=\frac{1}{2}\tilde{\omega}\left([F]^T[K][\delta x_n]+[\delta x_n]^T[K][F]\right)=\tilde{\omega}L_1=-\tilde{\omega}([B][\tilde{x}]+[b])^T[Q][\delta x_n], \tag{28}$$

which is satisfied if

$$[F]=-[K]^{-1}[Q]([B][\tilde{x}]+[b]). \tag{29}$$

According to (23) the update law of the estimate becomes

$$[\delta\dot{x}_n]=\tilde{\omega}[F]=-[K]^{-1}[Q]([B][\tilde{x}]+[b])\tilde{\omega} \tag{30}$$

This update law, together with the control law (26) assures the stability of the extended system

$$\begin{bmatrix}[\dot{\tilde{x}}]\\ [\delta\dot{x}_n]\end{bmatrix}=\begin{bmatrix}[A] & [0]\\ [0] & [0]\end{bmatrix}\begin{bmatrix}[\tilde{x}]\\ [\delta x_n]\end{bmatrix}+\begin{bmatrix}[B][\tilde{x}]+[b]\\ -[K]^{-1}[Q]([B][\tilde{x}]+[b])\end{bmatrix}\tilde{\omega} \tag{31}$$

Substituting the matrices of the system (6) into (26) and using the notation (1) and (2) it results:

$$([B][\tilde{x}]+[b])=\begin{bmatrix}0&1&0&0&0&0\\-1&0&0&0&0&0\\0&0&0&1&0&0\\0&0&-1&0&0&0\\0&0&0&0&0&1\\0&0&0&0&-1&0\end{bmatrix}\begin{bmatrix}\tilde{i}_{Lsd}\\ \tilde{i}_{Lsq}\\ \tilde{u}_{Cpd}\\ \tilde{u}_{Cpq}\\ \tilde{i}_{Lisd}\\ \tilde{i}_{Lisq}\end{bmatrix}+\begin{bmatrix}I_{Lsqn}\\ -I_{Lsdn}\\ U_{Cpqn}\\ -U_{Cpdn}\\ I_{Lisqn}\\ -I_{Lisdn}\end{bmatrix}=\begin{bmatrix}\tilde{i}_{Lsq}+I_{Lsqn}\\ -\tilde{i}_{Lsd}-I_{Lsdn}\\ \tilde{u}_{Cpq}+U_{Cpqn}\\ -\tilde{u}_{Cpd}-U_{Cpdn}\\ \tilde{i}_{Lisq}+I_{Lisqn}\\ -\tilde{i}_{Lisd}-I_{Lisdn}\end{bmatrix}=\begin{bmatrix}i_{LsqT}\\ -i_{LsdT}\\ u_{CpqT}\\ -u_{CpdT}\\ i_{LisqT}\\ -i_{LisdT}\end{bmatrix} \tag{32}$$

Thus, the control law (26) becomes:

$$\tilde{\omega} = -\alpha \begin{pmatrix} i_{LsqT} L_s \left(i_{LsdT} - \tilde{I}_{Lsdn} \right) - i_{LsdT} L_s \left(i_{LsqT} - \tilde{I}_{Lsqn} \right) + u_{CpqT} C_p \left(u_{CpdT} - \tilde{U}_{Cpdn} \right) - \\ - u_{CpdT} C_p \left(u_{CpqT} - \tilde{U}_{Cpqn} \right) + i_{LisqT} L_{is} \left(i_{LisdT} - \tilde{I}_{Lisdn} \right) - i_{LisdT} L_{is} \left(i_{LisqT} - \tilde{I}_{Lisqn} \right) \end{pmatrix} \tag{33}$$

Assuming that $[K]$ is diagonal, with the diagonal elements equal to *k*, the update law of the estimate (30) becomes:

$$\begin{cases} \delta\dot{\tilde{I}}_{Lsdn} = -k^{-1} L_s i_{LsqT} \tilde{\omega} \\ \delta\dot{\tilde{I}}_{Lsqn} = k^{-1} L_s i_{LsdT} \tilde{\omega} \\ \delta\dot{\tilde{U}}_{Cpdn} = -k^{-1} C_p u_{CpqT} \tilde{\omega} \\ \delta\dot{\tilde{U}}_{Cpqn} = k^{-1} C_p u_{CpdT} \tilde{\omega} \\ \delta\dot{\tilde{I}}_{Lisdn} = -k^{-1} L_{is} i_{LisqT} \tilde{\omega} \\ \delta\dot{\tilde{I}}_{Lisqn} = k^{-1} L_{is} i_{LisdT} \tilde{\omega} \end{cases} \tag{34}$$

The estimated steady-state value of the variable x results:

$$\tilde{X}_n = \tilde{X}_{n0} + \int_0^t \delta\dot{\tilde{X}}_n dt, \cdots with\ \tilde{X}_{n0} = \tilde{X}_n \Big|_{t=0} \tag{35}$$

Fig. 7 shows the control structure of U_{Cp} load capacitor voltage, based on the control law (33). In this control structure, the circuit variables are estimated according to formulae (34) and (35), while the steady-state angular velocity estimate is updated by an integrator according to:

$$\tilde{\omega}_n = \tilde{\omega}_n \Big|_{t=0} + K_{iUCp} \int_0^t \left(U_{Cp} - U_{Cpref} \right) dt` , \tag{36}$$

valid for the upper frequency domain from *Fig. 1.b*.

Fig. 8 shows comparison of the inverter start-up (set value $U_{Cpref} = 300V$), according to the above adaptive control law and start-up with PI controller tuned for the nominal inductor parameters in the case when the inductor is almost "empty", i.e. its equivalent resistance is one third of the nominal value.

The nominal parameters of the resonant load are:
L_s= 20μH, L_{is}= 3.95μH, R_{is}= 0.03Ω, C_p= 63μF.

The amplitude of the inverter output voltage's fundamental is u_{i1max}=266V.
Fig. 9 shows the update process of the estimated steady-state values starting from the initial values:

$$\tilde{I}_{Lsdn0} = 121\,A, \tilde{I}_{Lsqn0} = -348\,A, \tilde{U}_{Cpdn0} = -243\,V, \tilde{U}_{Cpqn0} = -177\,V,$$
$$\tilde{I}_{Lisdn0} = -692\,A, \tilde{I}_{Lisqn0} = 770\,A, \omega_{n0} = 80{,}000\,rad/s .$$

These values belong to the $\omega_{n1} = 73062\, rad/s$ operating angular frequency for which $u_{Cp\max ref} = U_{Cpref} = 300\,V$ in case of nominal circuit parameters.

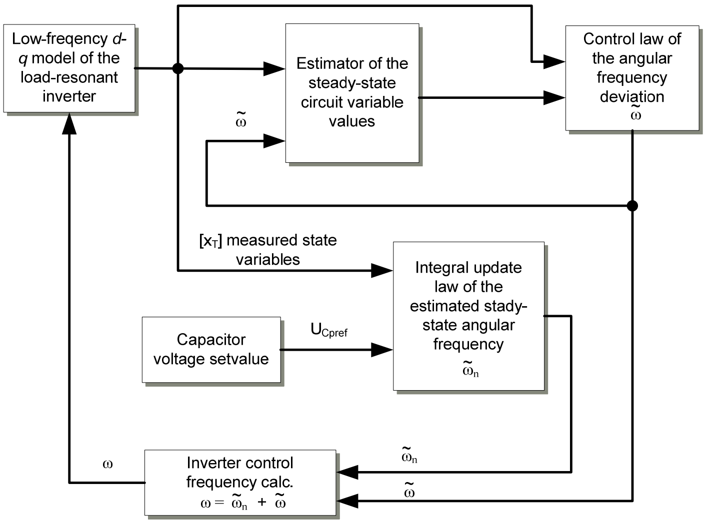
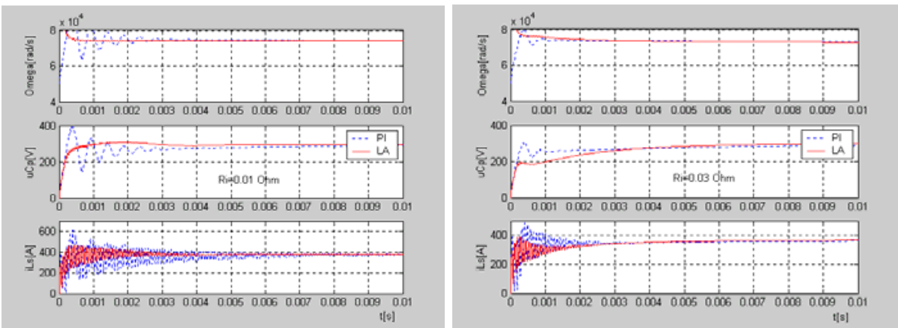


Figure 7: Control structure of the load capacitor tank voltage based on Lyapunov stability theory, with steady-state estimation.

Figures 8 and *9* show operation according to the proposed control law with controller parameters: $K = 0.02, \alpha = 1000, K_{iUCp} = 20000\, 1/Vs^2$.

The main practical difficulty is the measurement of the fundamental amplitudes and phases of three electrical quantities: u_{Cp}, i_{Ls}, i_{Lis}.



a) b)

Figure 8: Inverter start-up (R_{is}=0.01 Ω in case (a), R_{is}=0.03 Ω in case (b)) with Lyapunov-based control with steady-state estimation (LA), and PI control. In both cases the initial angular frequency is 80,000 rad/s.

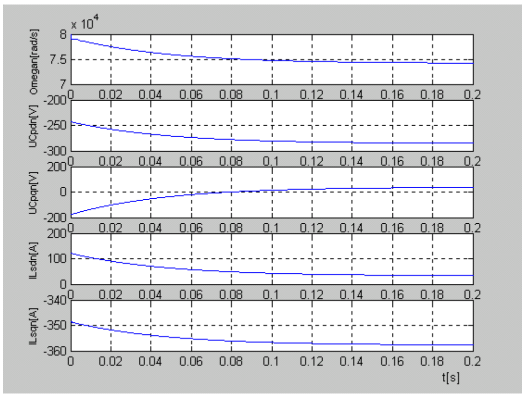


Figure 9 : Update process of the estimated steady-state values in case of R_{is}=0.01 Ω, much different from its "nominal" value.

6. Conclusions

A load-resonant inverter is a nonlinear system, which is difficult to control in case of high quality factor of the inductor. Lyapunov-based control methods have been proposed and demonstrated by simulation both for known and uncertain load parameters.

A possible further development consists in reduction of the number of measured state variables based on the circuit equations with "nominal" parameters.

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