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Contact warped product semi-slant submanifolds of $(LCS)_n$-manifolds

Shyamal Kumar Hui
Nikhil Banga Sikshan Mahavidyalaya
Bishnupur, Bankura – 722 122
West Bengal, India
email: shyamal_hui@yahoo.co.in

Mehmet Atceken
Gaziosmanpasa University
Faculty of Arts and Sciences,
Department of Mathematics
Tokat – 60250, Turkey
email: matceken@gop.edu.tr

Abstract. The present paper deals with a study of warped product submanifolds of $(LCS)_n$-manifolds and warped product semi-slant submanifolds of $(LCS)_n$-manifolds. It is shown that there exists no proper warped product submanifolds of $(LCS)_n$-manifolds. However we obtain some results for the existence or non-existence of warped product semi-slant submanifolds of $(LCS)_n$-manifolds.

1 Introduction

The notion of warped product manifolds were introduced by Bishop and O'Neill [3] and later it was studied by many mathematicians and physicists. These manifolds are generalization of Riemannian product manifolds. The existence or non-existence of warped product manifolds plays some important role in differential geometry as well as physics.

The notion of slant submanifolds in a complex manifold was introduced and studied by Chen [7], which is a natural generalization of both invariant and anti-invariant submanifolds. Chen [7] also found examples of slant submanifolds of complex Euclidean space C^2 and C^4. Then Lotta [9] has defined and

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studied of slant immersions of a Riemannian manifold into an almost contact metric manifold and proved some properties of such immersions. Also Cabrerizo et. al ([5], [6]) studied slant immersions in Sasakian and K-contact manifolds respectively. Again Gupta et. al [8] studied slant submanifolds of a Kenmotsu manifolds and obtained a necessary and sufficient condition for a 3-dimensional submanifold of a 5-dimensional Kenmotsu manifold to be minimal proper slant submanifold.

In 1994 Papaghuic [13] introduced the notion of semi-slant submanifolds of almost Hermitian manifolds. Then Cabrerizo et. al [4] defined and investigated semi-slant submanifolds of Sasakian manifolds. In this connection, it may be mentioned that Sahin [14] studied warped product semi-slant submanifolds of Kaehler manifolds. Also in [1], Atceken studied warped product semi-slant submanifolds in locally Riemannian product manifolds. Again Atceken [2] studied warped product semi-slant submanifolds in Kenmotsu manifolds and he has shown the non-existence cases of the warped product semi-slant submanifolds in a Kenmotsu manifold [2].

Recently Shaikh [15] introduced the notion of Lorentzian concircular structure manifolds (briefly, $(LCS)_n$-manifolds), with an example, which generalizes the notion of LP-Sasakian manifolds introduced by Matsumoto [10] and also by Mihai and Rosca [11]. Then Shaikh and Baishya ([17], [18]) investigated the applications of $(LCS)_n$-manifolds to the general theory of relativity and cosmology. The $(LCS)_n$-manifolds is also studied by Sreenivasa et. al [21], Shaikh [16], Shaikh and Binh [19], Shaikh and Hui [20] and others.

The object of the paper is to study warped product semi-slant submanifolds of $(LCS)_n$-manifolds. The paper is organized as follows. Section 2 is concerned with some preliminaries. Section 3 deals with a study of warped product submanifolds of $(LCS)_n$-manifolds. It is shown that there do not exist proper warped product submanifolds $N = N_1 \times_f N_2$ of a $(LCS)_n$-manifold M, where N_1 and N_2 are submanifolds of M. In section 4, we investigate warped product semi-slant submanifolds of $(LCS)_n$-manifolds and obtain many interesting results.

2 Preliminaries

An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type (0,2) such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to \mathbb{R}$ is a non-degenerate inner product of signature

$(-,+,\cdots,+)$, where T_pM denotes the tangent vector space of M at p and $\mathbb{R}$ is the real number space. A non-zero vector $v \in T_pM$ is said to be timelike (resp., non-spacelike, null, spacelike) if it satisfies $g_p(v,v) < 0$ (resp, ≤ 0, $= 0$, > 0) [12].

Definition 1 [15] *In a Lorentzian manifold* (M, g) *a vector field* P *defined by*

$$g(X,P) = A(X),$$

for any $X \in \Gamma(TM)$*, is said to be a concircular vector field if*

$$(\bar{\nabla}_X A)(Y) = \alpha\{g(X,Y) + \omega(X)A(Y)\}$$

where α *is a non-zero scalar and* ω *is a closed 1-form and* $\bar{\nabla}$ *denotes the operator of covariant differentiation with respect to the Lorentzian metric* g*.*

Let M be an n-dimensional Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1. \tag{1}$$

Since ξ is a unit concircular vector field, it follows that there exists a non-zero 1-form η such that for

$$g(X,\xi) = \eta(X), \tag{2}$$

the equation of the following form holds

$$(\bar{\nabla}_X \eta)(Y) = \alpha\{g(X,Y) + \eta(X)\eta(Y)\} \quad (\alpha \neq 0) \tag{3}$$

for all vector fields X, Y, where $\bar{\nabla}$ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfies

$$\bar{\nabla}_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{4}$$

ρ being a certain scalar function given by $\rho = -(\xi\alpha)$. Let us take

$$\phi X = \frac{1}{\alpha}\bar{\nabla}_X \xi, \tag{5}$$

then from (3) and (5) we have

$$\phi X = X + \eta(X)\xi, \tag{6}$$

from which it follows that ϕ is a symmetric (1,1) tensor and called the structure tensor of the manifold. Thus the Lorentzian manifold M together with the unit timelike concircular vector field ξ, its associated 1-form η and an (1,1) tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly, $(LCS)_n$-manifold) [15]. Especially, if we take $\alpha = 1$, then we can obtain the LP-Sasakian structure of Matsumoto [10]. In a $(LCS)_n$-manifold ($n > 2$), the following relations hold [15]:

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{7}$$

$$\phi^2 X = X + \eta(X)\xi, \tag{8}$$

$$S(X, \xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{9}$$

$$R(X, Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y], \tag{10}$$

$$R(\xi, Y)Z = (\alpha^2 - \rho)[g(Y, Z)\xi - \eta(Z)Y], \tag{11}$$

$$(\bar{\nabla}_X \phi)(Y) = \alpha\{g(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{12}$$

$$(X\rho) = d\rho(X) = \beta\eta(X), \tag{13}$$

$$R(X, Y)Z = \phi R(X, Y)Z + (\alpha^2 - \rho)\{g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\}\xi, \tag{14}$$

for all X, Y, $Z \in \Gamma(TM)$ and $\beta = -(\xi\rho)$ is a scalar function, where R is the curvature tensor and S is the Ricci tensor of the manifold.

Let N be a submanifold of a $(LCS)_n$-manifold M with induced metric g. Also let ∇ and $\nabla^\perp$ are the induced connections on the tangent bundle TN and the normal bundle $T^\perp N$ of N respectively. Then the Gauss and Weingarten formulae are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{15}$$

and

$$\bar{\nabla}_X V = -A_V X + \nabla^\perp_X V \tag{16}$$

for all $X, Y \in \Gamma(TN)$ and $V \in \Gamma(T^\perp N)$, where h and A_V are second fundamental form and the shape operator (corresponding to the normal vector field V) respectively for the immersion of N into M. The second fundamental form h and the shape operator A_V are related by [22]

$$g(h(X, Y), V) = g(A_V X, Y) \tag{17}$$

for any $X, Y \in \Gamma(TN)$ and $V \in \Gamma(T^\perp N)$.

For any $X \in \Gamma(TN)$, we may write

$$\phi X = EX + FX, \tag{18}$$

where EX is the tangential component and FX is the normal component of ϕX.
Also for any $V \in \Gamma(T^{\perp}N)$, we have

$$\phi V = BV + CV, \tag{19}$$

where BV and CV are the tangential and normal components of ϕV respectively. From (18) and (19) we can derive the tensor fields E, F, B and C are also symmetric. The covariant derivatives of the tensor fields of E and F are defined as

$$(\nabla_X E)(Y) = \nabla_X EY - E(\nabla_X Y), \tag{20}$$

$$(\bar{\nabla}_X F)(Y) = \nabla^{\perp}_X FY - F(\nabla_X Y) \tag{21}$$

for all X, $Y \in \Gamma(TN)$. The canonical structures E and F on a submanifold N are said to be parallel if $\nabla E = 0$ and $\bar{\nabla} F = 0$ respectively.

Throughout the paper, we consider ξ to be tangent to N. The submanifold N is said to be invariant if F is identically zero, i.e., $\phi X \in \Gamma(TN)$ for any $X \in \Gamma(TN)$. Also N is said to anti-invariant if E is identically zero, that is $\phi X \in \Gamma(T^{\perp}N)$ for any $X \in \Gamma(TN)$.

Furthermore for submanifolds tangent to the structure vector field ξ, there is another class of submanifolds which is called slant submanifold. For each non-zero vector X tangent to N at x, the angle $\theta(x)$, $0 \leq \theta(x) \leq \frac{\pi}{2}$ between ϕX and EX is called the slant angle or wirtinger angle. If the slant angle is constant, then the submanifold is also called the slant submanifold. Invariant and anti-invariant submanifolds are particular slant submanifolds with slant angle $\theta = 0$ and $\theta = \frac{\pi}{2}$ respectively. A slant submanifold is said to be proper slant if the slant angle θ lies strictly between 0 and $\frac{\pi}{2}$, i.e., $0 < \theta < \frac{\pi}{2}$ [5].

Lemma 1 [5] *Let* N *be a submanifold of a* $(LCS)_n$*-manifold* M *such that* ξ *is tangent to* N*. Then* N *is slant submanifold if and only if there exists a constant* $\lambda \in [0, 1]$ *such that*

$$E^2 = \lambda(I + \eta \otimes \xi). \tag{22}$$

Furthermore, if θ is the slant angle of N, then $\lambda = \cos^2\theta$.
Also from (22) we have

$$g(EX, EY) = \cos^2\theta[g(X, Y) + \eta(X)\eta(Y)], \tag{23}$$

$$g(FX, FY) = \sin^2\theta[g(X, Y) + \eta(X)\eta(Y)] \tag{24}$$

for any X, Y tangent to N.

The study of semi-slant submanifolds of almost Hermitian manifolds was introduced by Papaghuic [13], which was extended to almost contact manifold by Cabrerizo et. al [4]. The submanifold N is called semi-slant submanifold of M if there exist an orthogonal direct decomposition of TN as

$$TN = D_1 \oplus D_2 \oplus \{\xi\},$$

where D_1 is an invariant distribution, i.e., $\phi(D_1) = D_1$ and D_2 is slant with slant angle $\theta \neq 0$. The orthogonal complement of FD_2 in the normal bundle $T^{\perp}N$ is an invariant subbundle of $T^{\perp}N$ and is denoted by μ. Thus we have

$$T^{\perp}N = FD_2 \oplus \mu.$$

Similarly N is called anti-slant subbundle of M if D_1 is an anti-invariant distribution of N, i.e., $\phi D_1 \subset T^{\perp}N$ and D_2 is slant with slant angle $\theta \neq 0$.

3 Warped product submanifolds of $(LCS)_n$-manifolds

The notion of warped product manifolds were introduced by Bishop and O'Neill [3].

Definition 2 *Let* (N_1, g_1) *and* (N_2, g_2) *be two Riemannian manifolds and* f *be a positive definite smooth function on* N_1*. The warped product of* N_1 *and* N_2 *is the Riemannian manifold* $N_1 \times_f N_2 = (N_1 \times N_2, g)$*, where*

$$g = g_1 + f^2 g_2. \tag{25}$$

A warped product manifold $N_1 \times_f N_2$ is said to be trivial if the warping function f is constant.
More explicitely, if the vector fields X and Y are tangent to $N_1 \times_f N_2$ at (x, y) then

$$g(X, Y) = g_1(\pi_1 * X, \pi_1 * Y) + f^2(x) g_2(\pi_2 * X, \pi_2 * Y),$$

where π_i $(i = 1, 2)$ are the canonical projections of $N_1 \times N_2$ onto N_1 and N_2 respectively and * stands for the derivative map.

Let $N = N_1 \times_f N_2$ be warped product manifold, which means that N_1 and N_2 are totally geodesic and totally umbilical submanifolds of N respectively. For warped product manifolds, we have [3]

Proposition 1 *Let* $N = N_1 \times_f N_2$ *be a warped product manifold. Then*

(I) $\nabla_X Y \in TN_1$ *is the lift of* $\nabla_X Y$ *on* N_1

(II) $\nabla_U X = \nabla_X U = (X \ln f) U$

(III) $\nabla_U V = \nabla'_U V - g(U, V) \nabla \ln f$

for any X, $Y \in \Gamma(TN_1)$ *and* U, $V \in \Gamma(TN_2)$, *where* ∇ *and* ∇' *denote the Levi-Civita connections on* N_1 *and* N_2 *respectively.*

We now prove the following:

Theorem 1 *There exist no proper warped product submanifolds in the form* $N = N_T \times_f N_\perp$ *of a* $(LCS)_n$*-manifold* M *such that* ξ *is tangent to* N_T*, where* N_T *and* $N_\perp$ *are invariant and anti-invariant submanifolds of* M*, respectively.*

Proof. We suppose that $N = N_T \times_f N_\perp$ is a warped product submanifold of $(LCS)_n$-manifold M. For any $X \in \Gamma(TN_T)$ and $U, V \in \Gamma(TN_\perp)$, from Proposition 1 we have

$$\nabla_U X = \nabla_X U = (X \ln f)U. \tag{26}$$

On the other hand, by using (12) and (26) we have

$$\begin{aligned}(X \ln f)g(U,V) &= g(\nabla_U X, V) = g(\bar{\nabla}_U X, V) = g(\phi\nabla_U X, \phi V)\\ &= g(\bar{\nabla}_U \phi X - (\bar{\nabla}_U \phi)X, \phi V) = g(h(U,\phi X), \phi V) - \alpha\eta(X)g(U,\phi V)\\ &= g(h(U,\phi X), \phi V) = g(\bar{\nabla}_{\phi X} U, \phi V) = g(\phi\bar{\nabla}_{\phi X} U, V)\\ &= g(\bar{\nabla}_{\phi X}\phi U - (\bar{\nabla}_{\phi X}\phi)U, V) = g(\bar{\nabla}_{\phi X}\phi U, V)\\ &= -g(A_{\phi U}\phi X, V) = -g(h(\phi X, V), \phi U) = -g(\bar{\nabla}_V \phi X, \phi U)\\ &= -g(\bar{\nabla}_V X, U) = -g(\nabla_V X, U) = -(X \ln f)g(U,V).\end{aligned}$$

It follows that $X(\ln f) = 0$. So f is constant on N_T. Hence we get our desired assertion.

4 Warped product semi-slant submanifolds of $(LCS)_n$-manifolds

Let us suppose that $N = N_1 \times_f N_2$ be a warped product semi-slant submanifold of a $(LCS)_n$-manifold M. Such submanifolds are always tangent to the structure vector field ξ. If the manifolds N_θ and N_T (respectively $N_\perp$) are slant and invariant (respectively anti-invariant) submanifolds of a $(LCS)_n$-manifold M, then their warped product semi-slant submanifolds may be given by one of the following forms:
(i) $N_T \times_f N_\theta$ (ii) $N_\perp \times_f N_\theta$ (iii) $N_\theta \times_f N_T$ (iv) $N_\theta \times_f N_\perp$.

However, the existence or non-existence of a structure on a manifold is very important. Because the every structure of a manifold may not be admit. In this paper, we have researched cases that there exist no warped product semi-slant submanifolds in a $(LCS)_n$-manifold. Therefore we now study each of the above four cases and begin the following Theorem:

Theorem 2 *There exist no proper warped product semi-slant submanifold in the form* $N = N_T \times_f N_\theta$ *of a* $(LCS)_n$*-manifold* M *such that* ξ *is tangent to* N_T*, where* N_T *and* N_θ *are invariant and slant submanifolds of* M*, respectively.*

Proof. Let us assume that $N = N_T \times_f N_\theta$ is a proper warped product semi-slant submanifolds of a $(LCS)_n$-manifold M such that ξ is tangent to N_T. Then for any $X, \xi \in \Gamma(TN_T)$ and $U \in \Gamma(TN_\theta)$, from (5) and (15) we have

$$\bar{\nabla}_U \xi = \nabla_U \xi + h(U, \xi) = \alpha \phi U. \tag{27}$$

From the tangent and normal components of (27), respectively, we obtain

$$\xi(\ln f)U = \alpha EU \quad \text{and} \quad h(U, \xi) = \alpha FU. \tag{28}$$

On the other hand, by using (7) and (12), we have

$$\begin{aligned} (\bar{\nabla}_U \phi)\xi &= -\phi \bar{\nabla}_U \xi \\ \alpha U &= \phi(\xi(\ln f)U) + \phi h(U, \xi), \end{aligned}$$

that is,

$$B(U, \xi) + \xi(\ln f)EU = \alpha U \quad \text{and} \quad \xi(\ln f)FU + Ch(U, \xi) = 0. \tag{29}$$

Since $\Gamma(\mu)$ and $\Gamma(F(TN_\theta))$ are orthogonal subspaces, we can derive $\xi(\ln f)FU = 0$. So we conclude $\xi(\ln f) = 0$ or $FU = 0$. Here we have to show that FU for the proof. For this we assume that $FU \neq 0$.

Making use of (12), (15), (16) and (18), we obtain

$$\begin{aligned} (\bar{\nabla}_X \phi)U &= \bar{\nabla}_X \phi U - \phi \bar{\nabla}_X U \\ h(X, EU) - A_{FU}X + \nabla^{\perp}_X FU &= X(\ln f)FU + Bh(X, U) + Ch(X, U). \end{aligned} \tag{30}$$

Taking into account that the tangent components of (30) and making the necessary abbreviations, we get

$$A_{FU}X = -Bh(X, U). \tag{31}$$

With similar thoughts, we have

$$\begin{aligned} (\bar{\nabla}_U \phi)X &= \bar{\nabla}_U \phi X - \phi \bar{\nabla}_U X \\ \alpha \eta(X)U &= EX(\ln f)U + h(U, EX) - X(\ln f)EU - X(\ln f)FU \\ &- Bh(X, U) - Ch(X, U). \end{aligned} \tag{32}$$

From the normal components of (32), we arrive at

$$X(\ln f)FU = h(U, EX) - Ch(U, X). \tag{33}$$

Thus by using (31) and (33), we conclude

$$\begin{aligned} X(\ln f)g(FU, FU) &= g(h(U, EX), FU) = g(A_{FU}EX, U) = -g(Bh(EX, U), U) \\ &= -g(\phi h(EX, U), U) = -g(h(U, EX), FU) \\ &= -X(\ln f)g(FU, FU). \end{aligned}$$

This tell us that $X(\ln f) = 0$, that is, f is a constant function N_T because FU is a non-null vector field and N_θ is a proper slant submanifold.

Theorem 3 *There exist no proper warped product semi-slant submanifolds in the form* $N = N_\perp \times_f N_\theta$ *of a* $(LCS)_n$*-manifold* M *such that* ξ *is tangent to* N*, where* $N_\perp$ *and* N_θ *are anti-invariant and proper slant submanifolds of* M *respectively.*

Proof. Let $N = N_\perp \times_f N_\theta$ be a proper warped product semi-slant submanifold of a $(LCS)_n$-manifold M such that ξ is tangent to N. If ξ is tangent to $\Gamma(TN_\theta)$, then for any $X \in \Gamma(TN_\theta)$ and $U \in \Gamma(TN_\perp)$, from (5) and (15), we have

$$\bar{\nabla}_U \xi = \nabla_U \xi + h(U, \xi) = \alpha\phi U, \tag{34}$$

which is equivalent to $U(\ln f)\xi = 0$ because $\xi \neq 0$. So f is a constant function on $N_\perp$.

On the other hand, if $\xi \in \Gamma(TN_\perp)$, from (5) and (15), we reach

$$\begin{aligned} \bar{\nabla}_X \xi &= \nabla_X \xi + h(X, \xi) \\ \alpha\phi X &= \xi(\ln f)X + h(X, \xi), \end{aligned}$$

that is,

$$\alpha EX = \xi(\ln f)X \quad \text{and} \quad \alpha FX = h(X, \xi). \tag{35}$$

Furthermore, since $\phi\xi = 0$, by direct calculations, we obtain

$$\begin{aligned} (\bar{\nabla}_X \phi)\xi &= -\phi(\bar{\nabla}_X \xi) \\ \alpha X &= \xi(\ln f)EX + \xi(\ln f)FX + Bh(X, \xi) + Ch(X, \xi). \end{aligned}$$

It follows that

$$\alpha X = \xi(\ln f)EX + Bh(X, \xi) \quad \text{and} \quad \xi(\ln f)FX = -Ch(X, \xi). \tag{36}$$

By virtue of (36), we conclude

$$\xi(\ln f)g(FX,FX) = \sin^2\theta\xi(\ln f)g(X,X) = -g(Ch(X,\xi),FX) = 0,$$

which follows $\xi(\ln f) = 0$ or $\sin^2\theta g(X,X) = 0$. Here if $\xi(\ln f) \neq 0$ and $\sin^2\theta g(X,X) = 0$, the proof is obvious. Otherwise, making use of (36), we conclude that

$$\alpha g(X,X) = g(Bh(X,\xi),X) = 0.$$

Consequently, we can easily to see that $\alpha = 0$. This is a contradiction because the ambient space M is a $(LCS)_n$-manifold. Thus the proof is complete.

Theorem 4 *There exist no proper warped product semi-slant submanifolds in the form* $N_\theta \times_f N_T$ *in* $(LCS)_n$*-manifold* M *such that* ξ *tangent to* N_T*, where* N_θ *and* N_T *are proper slant and invariant submanifolds of* M.

Proof. Let $N = N_\theta \times_f N_T$ be warped product semi-slant submanifolds in a $(LCS)_n$-manifold M such that ξ is tangent to N_T. Then for any $\xi, X \in \Gamma(TN_T)$ and $U \in \Gamma(TN_\theta)$, taking account of relations (12), (15), (16), (18) and (19) and Proposition 1, we have

$$\begin{aligned} (\bar{\nabla}_U\phi)X &= \bar{\nabla}_U\phi X - \phi\bar{\nabla}_U X \\ \alpha\eta(X)U &= h(U,EX) - Bh(U,X) - Ch(U,X), \end{aligned}$$

which implies that

$$\alpha\eta(X)U = -Bh(U,X) \quad \text{and} \quad h(U,EX) = Ch(U,X). \tag{37}$$

In the same way, we have

$$\begin{aligned} (\bar{\nabla}_X\phi)U &= \bar{\nabla}_X\phi U - \phi\bar{\nabla}_X U \\ -A_{FU}X + \nabla^{\perp}_X FU + h(X,EU) &= Bh(X,U) + Ch(X,U), \end{aligned}$$

from here

$$Bh(X,U) = -A_{FU}X + EU(\ln f)X - U(\ln f)EX \tag{38}$$

and

$$\nabla^{\perp}_X FU = Ch(X,U) - h(X,EU). \tag{39}$$

Taking inner product both of sides of (37) with $V \in \Gamma(TN_\theta)$ and also using (38), we arrive at

$$\begin{aligned}\alpha\eta(X)g(U,V) &= -g(Bh(U,X),V) = -g(\phi h(U,X),V) = -g(h(U,X),\phi V)\\ &= -g(h(U,X),FV) = -g(A_{FV}X,U) = g(Bh(X,V),U)\\ &= -\alpha\eta(X)g(U,V).\end{aligned}$$

Here for $X = \xi$, we obtain $\alpha g(U,V) = 0$. Because the ambient space M is a $(LCS)_n$-manifold and N_θ is a proper slant submanifold, this also tells us the accuracy of the statement of the theorem.

Theorem 5 *There exist no proper warped product semi-slant submanifolds in the form* $N = N_\theta \times_f N_\perp$ *in a* $(LCS)_n$*-manifold* M *such that* ξ *tangent to* N_θ*, where* N_θ *and* $N_\perp$ *are proper slant and anti-invariant submanifolds of* M*, respectively.*

Proof. Let us assume that $N = N_\theta \times_f N_\perp$ be a proper warped product semi-slant submanifold in the $(LCS)_n$-manifold M such that ξ is tangent to N_θ. Then for $X \in \Gamma(TN_\theta)$ and $U \in \Gamma(TN_\perp)$, we have

$$\begin{aligned}(\bar{\nabla}_X\phi)U &= \bar{\nabla}_X\phi U - \phi\bar{\nabla}_X U\\ -A_{FU}X + \nabla^\perp_X FU &= \phi\nabla_X U + \phi h(X,U),\end{aligned}$$

which follows that

$$A_{FU}X = -Bh(X,U) \quad and \quad (\nabla_X F)U = Ch(X,U). \tag{40}$$

In the same way, we have

$$(\bar{\nabla}_U\phi)X = \bar{\nabla}_U\phi X - \phi\bar{\nabla}_U X,$$

which also follow that

$$\alpha\eta(X)U = EX(\ln f)U - A_{FX}U - Bh(X,U), \tag{41}$$

$$\nabla^\perp_U FX = X(\ln f)FU + Ch(X,U) - h(U,EX). \tag{42}$$

From (41), we can derive

$$g(h(U,X),FX) = g(h(U,X),FU) = 0. \tag{43}$$

Taking $X = \xi$ in (42), we have $\xi(\ln f)FU = -Ch(X,\xi)$, that is, $\xi(\ln f)FU = 0$. Let $X = \xi$ be in (41), then we get

$$\alpha U = Bh(U,\xi). \tag{44}$$

Taking the inner product of the both sides of (44) by $U \in \Gamma(TN_\perp)$, and using (43) we conclude

$$\alpha g(U, U) = g(Bh(U, \xi), U) = g(h(U, \xi), FU) = 0, \tag{45}$$

which implies that $\alpha = 0$. This is impossible because the ambient space is a $(LCS)_n$-manifold. Hence the proof is complete.

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