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A Liquid Metal Flow Between Two Coaxial Cylinders System

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Abstract: The Taylor-Couette flow between two rotating coaxial cylinders remains an ideal tool for understanding the mechanism of the transition from laminar to turbulent regime in rotating flow for the scientific community. We present for different Taylor numbers a set of three-dimensional numerical investigations of the stability and transition from Couette flow to Taylor vortex regime of a viscous incompressible fluid (liquid sodium) between two concentric cylinders with the inner one rotating and the outer one at rest. We seek the onset of the first instability and we compare the obtained results for different velocity rates. We calculate the corresponding Taylor number in order to show its effect on flow patterns and pressure field.

Keywords: Taylor-Couette, rotating cylinders, liquid metal flow.

1. Introduction

The fluid contained in the rotor-stator gap, driven by the inner rotating cylinder is called Couette flow, or commonly, Taylor-Couette flow [1-4]. The rotation of the inner cylindrical wall produces the least stable situation, because it generates a centrifugal force which ejects the fluid towards the outer wall, thus destabilizing the flow. A theoretical formalization of this problem has developed for the case of inviscid fluids [2, 3].

In 1923, in a seminal paper, Taylor [4] combined the theoretical and experimental approaches and studied the linear stability of a viscous fluid, as shown by the Couette profile, valid for low speeds. When the angular velocity of the inner cylinder is increased above a certain threshold, the circular Couette flow becomes unstable and toroidal vortices flow known as Taylor vortex flow are occurring Taylor has shown that the stability calculation methods and the assumed boundary conditions give results that exactly match the experimental data. Since Taylor's famous article [4], numerous experimental, theoretical and numerical studies on the presence and evolution of Taylor's vortices have been undertaken and extended to other geometries [5-12]. The subject of liquid metal flow between two coaxial, independently rotating cylinders is important for the understanding of nonlinear phenomena of the rotating flows. These are ubiquitous in nature and technology contexts such as stellar interiors, accretion discs, the Earth's core, crystal growth furnaces intended for the ingots production according to the Kyropoulos or Czochralski techniques, and rotating machinery such as electric motors, pumps, rotating blade couplers. Moreover, the liquid sodium is widely used as a primary coolant for nuclear fusion reactors due to the physical and chemical features, high heat capacity and high thermal conductivity, that allow it to be used as a primary coolant for nuclear fusion reactors. In this paper, the focus is on the transition of Couette flow to Taylor vortex for different Taylor numbers. The considered fluid is the liquid sodium, which has many applications in industrial and engineering processes.

2. Modelling

The main objective of this work is to investigate numerically and to highlight the effect of the Taylor number values on the velocity field of the liquid sodium, which is an incompressible, inviscid and Newtonian liquid, contained in a small cylindrical gap with a large aspect ratio. The outer cylinder (stator), with $R_2 = 28.5\ mm$ is maintained stationary, and the inner one (rotor) with radius R1 = $23.65\ mm$ is rotating. The gap ratio δ is given by ($\delta = d / R_1$) where $d = 4.85\ mm$ is the gap between the cylinders ($d = R_2 - R_1$), as shown in *Fig. 1*. The height of the considered Taylor-Couette system is $H = 155\ mm$ as shown in *Fig. 1*. Therefore, this flow system is governed by the following parameters: radii ratio $R_1 / R_2 = 0.8$, which corresponds to a small annular gap, and a large aspect ratio $H / d = 31.9$. The inner cylinder rotates at angular velocity Ω which is increased stepwise during the computation. Different flow structures are then found. The first pattern corresponds to the circular Couette flow followed by the Taylor-vortex flow (TVF). TVF appears at a critical value Tc of the Taylor number defined as $Ta = Re.\delta^{1/2}$. Reynolds number is given by $Re = \rho R_1 \Omega d / \mu$. The working fluid is liquid sodium. It has constant density and viscosity $\rho = 920\ kg.m^{-3}$ and $\mu = 6.53\ 10^{-4}\ kg.m^{-1}\ s^{-1}$, respectively. The governing equations for the flow system are the conservation equations, namely, the continuity and Navier-Stokes equations. The cylindrical gap is conveniently described by r, θ and z, denoting the usual cylindrical coordinates the radial, azimuthal and axial directions respectively, as shown in *Fig. 1*. Further, we denote the velocity components in the increasing r, θ and z directions V_r,

V_θ and V_z respectively, and the pressure P. The governing equations can be written as,

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(V_\theta)+\frac{\partial}{\partial z}(V_Z)=0 \quad (1)$$

$$V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}+V_Z\frac{\partial V_r}{\partial z}-\frac{V_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\left\{\nabla^2 V_r-\frac{V_r}{r^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right\} \quad (2)$$

$$V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+V_Z\frac{\partial V_\theta}{\partial z}+\frac{V_r V_\theta}{r}=-\frac{1}{\rho}\frac{1}{r}\frac{\partial P}{\partial \theta}+\nu\left\{\nabla^2 V_\theta-\frac{V_\theta}{r^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right\} \quad (3)$$

$$V_r\frac{\partial V_z}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}+V_Z\frac{\partial V_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\nu\nabla^2 V_z \quad (4)$$

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2}, \quad (5)$$

where ρ and ν denote the liquid sodium density and kinematic viscosity respectively.

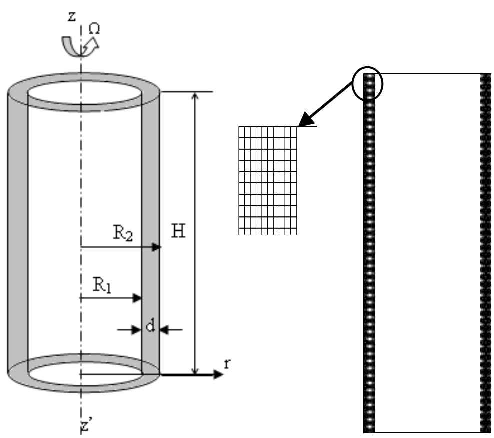


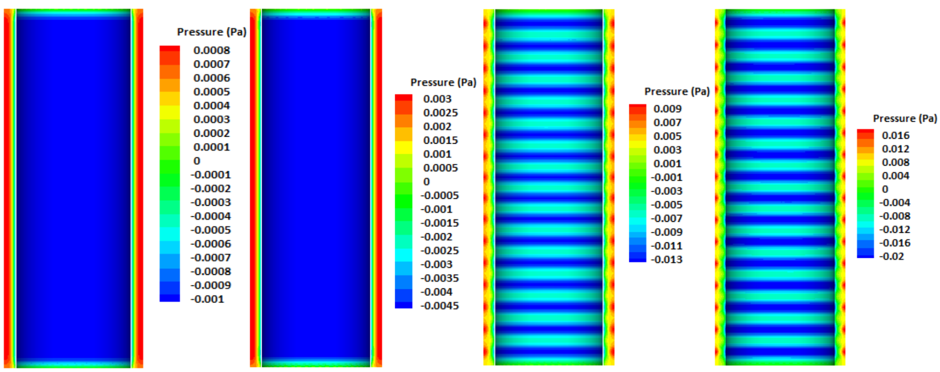
Figure 1.: Flow system, notations and used meshes.

In this paper, the focus is on the understanding of the Taylor number *Ta* effect on the distribution of velocity field in liquid confined between two coaxial cylinders. The Taylor number quantifies the ratio between centrifugal and viscous forces acting on liquid. Above the critical Taylor number value, centrifugal forces exceed viscous forces, and the flow becomes unstable. There are various forms of the Taylor number and the choice varies from one investigation to another, and it is used as a criterion of the stability of flow. In this work, the inner cylinder is rotating and the outer cylinder is fixed. At the wall, the flow has zero axial and radial velocity, while it is characterized by a tangential velocity component. Therefore, the associated non-slip boundary conditions are assumed. The radial, tangential and axial velocity components at the inner, outer walls and end-plates are given as $V_r\ (r = R_1$ or $r = R_2) = 0$, $V_r\ (Z = 0$ or $Z = H) = 0$, $V_\theta\ (r = R_1) = \Omega R_1$, $V_\theta\ (r = R_2) = 0$, $V_\theta\ (Z = 0 \text{ or } Z = H) = 0$, $V_z\ (r = R_1 \text{ or } r = R_2) = 0$ and $V_z\ (Z = 0 \text{ or } r = H) = 0$, respectively.

The problem is solved using the code ANSYS®Fluent 16.0 package based on the finite volume method. The momentum discretization has been performed with a QUICK scheme. The PISO algorithm (Pressure Implicit with Splitting of Operator) is applied for pressure-velocity coupling and the pressure-based solver has been discretized with PRESTO scheme (PREssure STaggering Option). The Green-Gauss cell-based method has been used for gradients evaluation. The convergence is handled by monitoring residuals of continuity and momentum equations, while the residuals are set to 10^{-6}. The used mesh is regular and it has about one million nodes. In *Fig. 1*, the structure of the mesh employed in the three-dimensional simulations is given.

3. Results and discussion

The stable laminar regime which is the basic flow existing in the absence of any disturbance.



Ta=18.29 *Ta=36.59* *Ta=45.88* *Ta=73.18*

Figure 2: Computed pressure for different Taylor numbers.

This can be described as a homogeneous movement with a high degree of symmetry throughout the fluid, which is characterized by the occurrence of two cells, one located at the lower edge and the other at the upper edge. Then, the speed of rotation is slightly increased and then the formation of Taylor vortices occurs, which propagate from the edges towards the middle until the complete appearance of the cells in the whole cylinder. The appearance of the first instability of Taylor-Couette consists of the occurrence of a periodic axial stationary wave associated with the base flow.

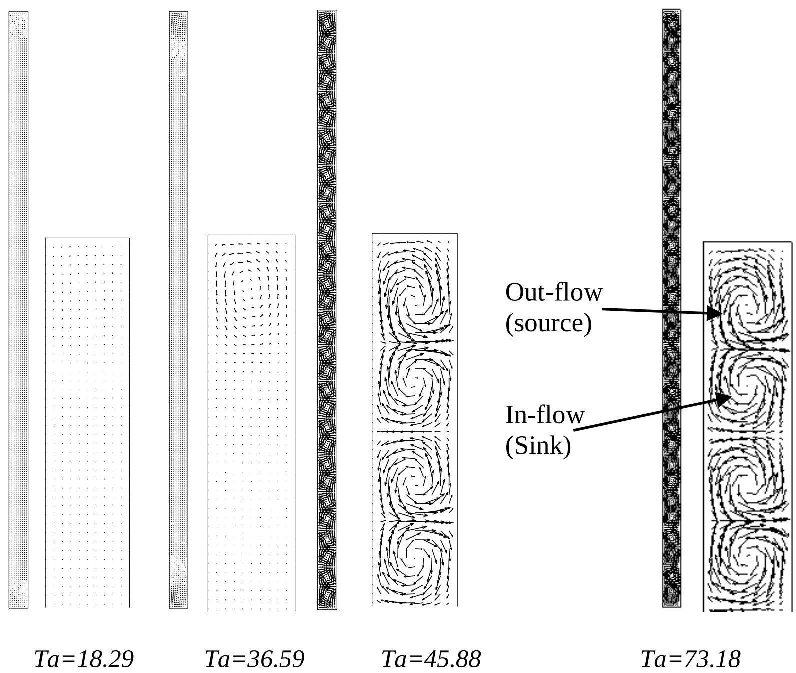


Figure 3: (Vr,Vz) components for different Taylor numbers.

The inner cylinder rotates counterclockwise as shown in *Fig. 1*, and the upper vortex rotates in the same direction as shown in *Fig. 3*. In succession, more vortices are added in the cylindrical gap, and the flow is organized in the form of rolls or Taylor vortices. These vortices travel along and around the inner cylinder. During the rotation flow, the source and the sink zones appear. *Fig. 4* depicts the axial, radial and tangential components of the flow in horizontal right plane. The velocity field V of the fluid is time-independent until the Taylor number exceeds a critical value $Ta=41.35$. This value is found to be in a good agreement with experimental and theoretical results found before [5, 10-12].

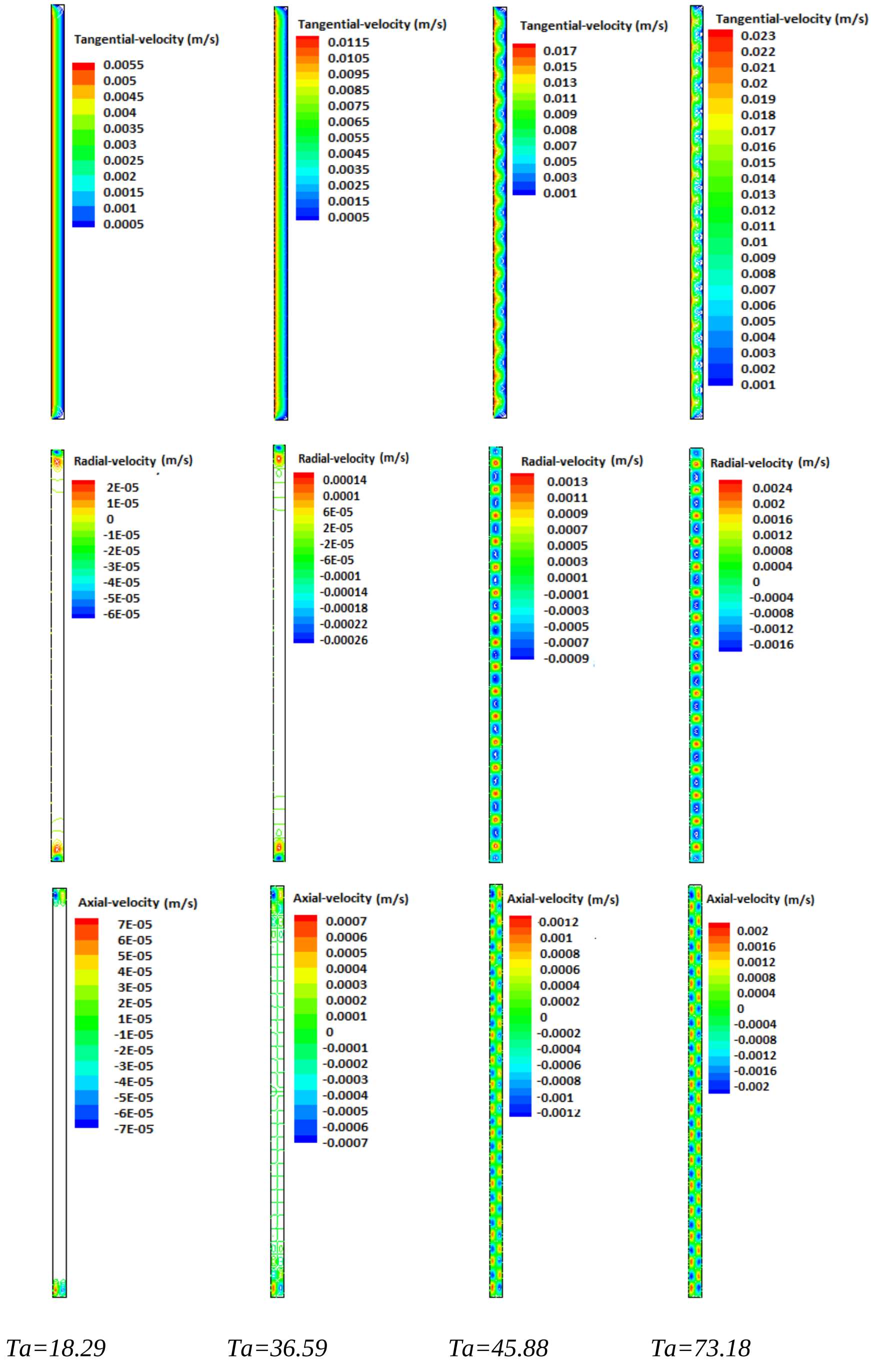


Figure 4: Velocity components for different Taylor numbers.

No-slip conditions imposed on the endplates of the system are responsible for the reducing of the azimuthal velocity component. The velocity components depend on Taylor Ta number values.

3. Conclusion

The first hydrodynamic instability of sodium liquid flow contained between two infinite coaxial cylinders, where the outer cylinder is stationary and the inner one is rotating was investigated numerically using the code ANSYS®Fluent 16.0 package based on the finite volume method. For very low Taylor numbers, the results show that the basic flow is stationary, axisymmetric invariant by vertical translation, and no cells are formed in the gap. By increasing the Taylor number, two first cells appeared close to each end plates, called Eckman's vortices. Beyond a threshold value, it is observed that this basic flow becomes unstable. For large Taylor numbers, the contra-rotating rollers in the liquid sodium cylindrical gap flow are appearing, and extend all-around of the inner cylinder. The tangential velocity values are larger than radial and axial velocity components. The results seem satisfactory regarding the hydrodynamic instability behavior, in agreement with experimental data found in the literature [13].

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