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On k-semi-centralizing maps of generalized matrix algebras

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Abstract. Let $\mathfrak{G} = \mathfrak{G}(\mathrm{A}, \mathrm{M}, \mathrm{N}, \mathrm{B})$ be a generalized matrix algebra over a commutative ring with unity. In the present article, we study k-semi-centralizing maps of generalized matrix algebras.

1 Historical development

Several authors studied commuting, centralizing and related maps on different rings and algebras see [1, 5–12, 15, 17, 19–21] and references therein. The study of centralizing mappings was initiated by a well known theorem due to Posner [16] which states that "the existence of a nonzero centralizing derivation on a prime ring $\mathfrak{R}$ must be commutative." In [14] Mayne investigated

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centralizing automorphisms of prime rings and proved that "if $\mathfrak{R}$ is a prime ring with a nontrivial centralizing automorphism, then $\mathfrak{R}$ is a commutative integral domain." These results due to Posner [16] and Mayne [14] have been extended by many authors in different ways (see [3,15,19–21] and in their existing refernces). In [15] Miers proved theorems for certain centralizing mappings of C^*-algebras and von Neumann algebras. Brešar [5] described that "all additive centralizing mappings f on prime rings $\mathfrak{R}$ of characteristic different from two has the form $f(x) = \lambda x + \xi(x)$, where λ is an element from the extended centroid of $\mathfrak{R}$ and ξ is an additive mapping from $\mathfrak{R}$ into the extended centroid of $\mathfrak{R}$." Also, Bell and Lucier investigated some results concerning skew commuting and skew centralizing additive maps in [3].

Cheung [7] initiated the study of linear commuting maps on matrix algebras and proved that "every commuting map on triangular algberas has proper form." Inspired by this result, Xiao and Wei in [21] described the general form of commuting maps on generalized matrix algebras and point out various related applications. Also, Li and Wei [13] proved that "any skew-commuting map on a class of generalized matrix algebras is zero and any semi-centralizing derivation on a generalized matrix algebra is zero."

Beidar [2] studied k-commuting maps in prime rings by applying the idea of functional identities in rings. Du and Wang [8] proved that "under certain conditions, each k-commuting mapping on a triangular algebra is proper." Recently, Li et al. [12] studied k-commuting mappings of generalized matrix algebras and determined the general form of arbitrary k-commuting mapping of a generalized matrix algebra. Now it is natural problem to study the k-semi-centralizing maps on matrix algebras.

Influenced by above stated references, in this article, we find out the structure of k-semi-centralizing maps on generalized matrix algebra under certain restrictions. Also, we prove that every k-centralizing map has the proper form on generalized matrix algebras. Moreover, we discuss an important result of this paper which states that every k-semi centralizing (commuting) derivation on a 2-torsion free generalized matrix algebra becomes zero. Lastly, we point out some direct consequences of our results.

2 Basic definitions & preliminaries

Let $\mathfrak{R}$ be a commutative ring with unity. An $\mathfrak{R}$-algebra $\mathfrak{G}$ denoted by the set

$$\mathfrak{G} = \mathfrak{G}(A, M, N, B) = \left\{ \begin{bmatrix} a & m \\ n & b \end{bmatrix} \middle| \, a \in A, m \in M, n \in N, b \in B \right\}$$

is said to be generalized matrix algebra under matrix like multiplication and usual matrix addition, if $(A, B, M, N, \xi_{MN}, \Omega_{NM})$ is a Morita context and either $M \neq 0$ or $N \neq 0$. A *Morita context* $(A, B, M, N, \xi_{MN}, \Omega_{NM})$ consisting of two unital $\mathfrak{R}$-algebras A and B, two bimodules (A, B)-bimodule M and (B, A)-bimodule N, and two bimodule homomorphisms called the bilinear pairings $\xi_{MN} : M \underset{B}{\otimes} N \longrightarrow A$ and $\Omega_{NM} : N \underset{A}{\otimes} M \longrightarrow B$ which satisfies the following commutative diagrams:

$$\begin{array}{ccc} M \underset{B}{\otimes} N \underset{A}{\otimes} M & \xrightarrow{\xi_{MN}\otimes I_M} & A \underset{A}{\otimes} M \\ \Big\downarrow{\scriptstyle I_M\otimes\Omega_{NM}} & & \Big\downarrow{\scriptstyle \cong} \\ M \underset{B}{\otimes} B & \xrightarrow{\cong} & M \end{array} \quad \text{and} \quad \begin{array}{ccc} N \underset{A}{\otimes} M \underset{B}{\otimes} N & \xrightarrow{\Omega_{NM}\otimes I_N} & B \underset{B}{\otimes} N \\ \Big\downarrow{\scriptstyle I_N\otimes\xi_{MN}} & & \Big\downarrow{\scriptstyle \cong} \\ N \underset{A}{\otimes} A & \xrightarrow{\cong} & N\,. \end{array}$$

More precisely, an $\mathfrak{R}$-algebra generated in this way is called as *generalized matrix algebra* of order 2 which was first introduced by Sands in [18]. $\mathfrak{G}$ becomes an upper triangular algebra provided $N = 0$ and $\mathfrak{G}$ degenerates a lower triangular algebra provided $M = 0$. Both upper and lower triangular algebras are collectively known as triangular algebras.

The center of $\mathfrak{G}$ is

$$\mathfrak{Z}(\mathfrak{G}) = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \,\middle|\, am = mb, na = bn \text{ for all } m \in M, n \in N \right\}.$$

Indeed $\mathfrak{Z}(\mathfrak{G})$ is a set diagonal matrices $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, where $a \in \mathfrak{Z}(A), b \in \mathfrak{Z}(B)$ and $am = mb, na = bn$ for all $m \in M$, $n \in N$. Also, if M is faithful left A-module and right B-module, then the condition $a \in \mathfrak{Z}(A), b \in \mathfrak{Z}(B)$ is superfluous and can be removed. Define two natural projections $\pi_A : \mathfrak{G} \to A$ and $\pi_B : \mathfrak{G} \to B$ by $\pi_A\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = a$ and $\pi_B\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = b$. Moreover, $\pi_A(\mathfrak{Z}(\mathfrak{G})) \subseteq \mathfrak{Z}(A)$ & $\pi_B(\mathfrak{Z}(\mathfrak{G})) \subseteq \mathfrak{Z}(B)$ and there exists a unique algebraic isomorphism $\xi : \pi_A(\mathfrak{Z}(\mathfrak{G})) \to \pi_B(\mathfrak{Z}(\mathfrak{G}))$ such that $am = m\xi(a)$ and $na = \xi(a)n$ for all $a \in \pi_A(\mathfrak{Z}(\mathfrak{G})), m \in M$ and $n \in N$.

Let 1_A (resp.1_B) be the identity of the algebra A (resp.B) and let I be the identity of generalized matrix algebra $\mathfrak{G}$, $e = \begin{bmatrix} 1_A & 0 \\ 0 & 0 \end{bmatrix}$, $f = I - e =$

$\begin{bmatrix} 0 & 0 \\ 0 & 1_B \end{bmatrix}$ and $\mathfrak{G}_{11} = e\mathfrak{G}e$, $\mathfrak{G}_{12} = e\mathfrak{G}f$, $\mathfrak{G}_{21} = f\mathfrak{G}e$, $\mathfrak{G}_{22} = f\mathfrak{G}f$. Thus $\mathfrak{G} = e\mathfrak{G}e + e\mathfrak{G}f + f\mathfrak{G}e + f\mathfrak{G}f = \mathfrak{G}_{11} + \mathfrak{G}_{12} + \mathfrak{G}_{21} + \mathfrak{G}_{22}$ where $\mathfrak{G}_{11}$ is subalgebra of $\mathfrak{G}$ isomorphic to A, $\mathfrak{G}_{22}$ is subalgebra of $\mathfrak{G}$ isomorphic to B, $\mathfrak{G}_{12}$ is $(\mathfrak{G}_{11}, \mathfrak{G}_{22})$-bimodule isomorphic to M and $\mathfrak{G}_{21}$ is $(\mathfrak{G}_{22}, \mathfrak{G}_{11})$-bimodule isomorphic to N. Also, $\pi_A(\mathfrak{Z}(\mathfrak{G}))$ and $\pi_B(\mathfrak{Z}(\mathfrak{G}))$ are isomorphic to $e\mathfrak{Z}(\mathfrak{G})e$ and $f\mathfrak{Z}(\mathfrak{G})f$ respectively. Then there is an algebra isomorphisms $\xi : e\mathfrak{Z}(\mathfrak{G})e \to f\mathfrak{Z}(\mathfrak{G})f$ such that $am = m\xi(a)$ and $na = \xi(a)n$ for all $m \in e\mathfrak{G}f$ and $n \in f\mathfrak{G}e$.

Let $\mathfrak{R}$ be a commutative ring with unity and $\mathcal{A}$ be an $\mathfrak{R}$-algebra. $\mathfrak{Z}(\mathcal{A})$ denote the center of $\mathcal{A}$ and define $\mathfrak{Z}(\mathcal{A})_k$ by $\{a \in \mathcal{A} \mid [a, y]_k = 0 \ \forall\, y \in \mathcal{A}\}$. In particular $\mathfrak{Z}(\mathcal{A})_1 = \mathfrak{Z}(\mathcal{A})$. For arbitrary elements $x, y \in \mathcal{A}$, we denote $[x, y]_0 = x$, $[x, y]_1 = xy - yx$, and inductively $[x, y]_k = [[x, y]_{k-1}, y]$, where $k > 0$ is a fixed positive integer. Also, denote $x \circ_0 y = x$, $x \circ_1 y = xy + yx$ and $x \circ_k y = (x \circ_{k-1} y) \circ_1 y$ for all $x, y \in \mathcal{A}$. An $\mathfrak{R}$-linear map $g : \mathcal{A} \to \mathcal{A}$ is said to semi-centralizing if $[g(x), x] \in \mathfrak{Z}(\mathcal{A})$ or $g(x) \circ x \in \mathfrak{Z}(\mathcal{A})$ for all $a \in \mathcal{A}$. Particularly, g is said to be centralizing if $[g(x), x] \in \mathfrak{Z}(\mathcal{A})$ and g is said to be skew centralizing if $g(x) \circ x \in \mathfrak{Z}(\mathcal{A})$ for all $x \in \mathcal{A}$. In general, for positive integer $k > 0$, an $\mathfrak{R}$-linear map $g : \mathcal{A} \to \mathcal{A}$ is said to k-semi-centralizing if $[g(x), x]_k \in \mathfrak{Z}(\mathcal{A})$ or $g(x) \circ_k x \in \mathfrak{Z}(\mathcal{A})$ for all $a \in \mathcal{A}$. In particular, g is said to be k-centralizing if $[g(x), x]_k \in \mathfrak{Z}(\mathcal{A})$ and g is said to be k-skew centralizing if $g(x) \circ_k x \in \mathfrak{Z}(\mathcal{A})$ for all $x \in \mathcal{A}$. Further, for positive integer $k > 0$, an $\mathfrak{R}$-linear map $g : \mathcal{A} \to \mathcal{A}$ is said to k-semi-commuting if $[g(x), x]_k = 0$ or $g(x) \circ_k x = 0$ for all $a \in \mathcal{A}$. In particular, g is said to be k-commuting if $[g(x), x]_k = 0$ and g is said to be k-skew commuting if $g(x) \circ_k x = 0$ for all $x \in \mathcal{A}$.

At this point, we shall mention some important results, which are essential for developing the proof of our main result:

Lemma 1 [12, Lemma 3.1] *Let n be a positive integer and $\mathfrak{R}$ be a unital associative ring. For a left $\mathfrak{R}$-module* M*, if $\alpha : \mathfrak{R} \to M$ is a mapping such that $\alpha(x + 1) = \alpha(x)$ and $x^n\alpha(x) = 0$ for all $x \in \mathfrak{R}$, then $\alpha = 0$. Similarly, for a right $\mathfrak{R}$-module* N*, a mapping $\beta : \mathfrak{R} \to N$ is zero if $\beta(x + 1) = \beta(x)$ and $\beta(x)x^n = 0$ for all $x \in \mathfrak{R}$.*

Lemma 2 [13, Proposition 4.2] *Let $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ be a generalized matrix algebra over a commutative ring $\mathfrak{R}$. An additive map $\Phi : \mathfrak{G} \to \mathfrak{G}$ is a derivation if and only if Φ has the following form*

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) - mn_0 - m_0n & am_0 + T_2(m) - m_0b \\ n_0a - bn_0 + V_3(n) & U_4(b) + nm_0 + n_0m \end{bmatrix},$$

where $a \in A$; $b \in B$; $m, m_0 \in M$; $n, n_0 \in N$ *and* $\Delta_1 : A \to A$, $T_2 : M \to$

M, $V_3 : N \to N$, $U_4 : B \to B$ *are* $\mathfrak{R}$*-linear maps satisfying the following conditions:*

1. Δ_1 *is a derivation of* A *and* $\Delta_1(mn) = T_2(m)n + mV_3(n)$;
2. U_4 *is a derivation of* B *and* $U_4(nm) = V_3(n)m + nT_2(m)$;
3. $T_2(am) = \Delta_1(a)m + aT_2(m)$ *and* $T_2(mb) = T_2(m)b + mU_4(b)$;
4. $V_3(na) = V_3(n)a + n\Delta_1(a)$ *and* $V_3(bn) = U_4(b)n + bV_3(n)$.

Lemma 3 [12, Theorem 3.5] *Let* $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ *be a generalized matrix algebra over a commutative ring* $\mathfrak{R}$ *and* $\Phi : \mathfrak{G} \to \mathfrak{G}$ *be a* k*-commuting map on* $\mathfrak{G}$*. If the following conditions are satisfied:*

1. $\mathfrak{Z}(A)_k = \pi_A(\mathfrak{Z}(\mathfrak{G}))$ *or* $[A, A] = A$;
2. $\mathfrak{Z}(B)_k = \pi_B(\mathfrak{Z}(\mathfrak{G}))$ *or* $[B, B] = B$;
3. *there exist* $m_0 \in M$, $n_0 \in N$ *such that*

$$\mathfrak{Z}(\mathfrak{G}) = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a \in \mathfrak{Z}(A), b \in \mathfrak{Z}(B), am_0 = m_0 b, n_0 a = bn_0 \right\},$$

then Φ *is proper i.e.,* Φ *has the form* $\Phi = \lambda + \xi$*, where* $\lambda \in \mathfrak{Z}(\mathfrak{G})$ *and* $\xi : \mathfrak{G} \to \mathfrak{Z}(\mathfrak{G})$ *is an* $\mathfrak{R}$*-linear mapping.*

3 Key content

In this section, we investigate the significant results of the article as follows:

Theorem 1 *Let* $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ *be a generalized matrix algebra over a commutative ring* $\mathfrak{R}$*. An* $\mathfrak{R}$*-linear map* $\Phi : \mathfrak{G} \to \mathfrak{G}$ *is a* k*-centralizing map on* $\mathfrak{G}$ *if* Φ *has the following form*

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) + \Delta_2(m) + \Delta_3(n) + \Delta_4(b) & T_2(m) \\ V_3(n) & U_1(a) + U_2(m) + U_3(n) + U_4(b) \end{bmatrix}, \qquad (\spadesuit)$$

where $a \in A$; $b \in B$; $m \in M$; $n \in N$ *and* $\Delta_1 : A \to A$, $\Delta_2 : M \to \mathfrak{Z}(A)_k$, $\Delta_3 : N \to \mathfrak{Z}(A)_k$, $\Delta_4 : B \to A$, $T_2 : M \to M$, $V_3 : N \to N$, $U_1 : A \to B$, $U_2 : M \to \mathfrak{Z}(B)_k$, $U_3 : N \to \mathfrak{Z}(B)_k$, $U_4 : B \to B$ *are* $\mathfrak{R}$*-linear maps satisfying the following conditions:*

1. Δ_1 *is* k*-commuting map of* A *and* $\Delta_1(1) \in \mathfrak{Z}(A)_k$;
2. U_4 *is* k*-commuting map of* B *and* $U_4(1) \in \mathfrak{Z}(B)_k$;
3. $[\Delta_4(b), a]_k \in \mathfrak{Z}(A)_k$ *and* $[U_1(a), b]_k \in \mathfrak{Z}(B)_k$;
4. $(\Delta_1(1) + \Delta_4(1) + 2\Delta_2(m))m = m(U_1(1) + U_4(1) + 2U_2(m))$;
5. $2T_2(m) = (\Delta_1(1) - \Delta_4(1))m - m(U_1(1) - U_4(1))$;
6. $n(\Delta_1(1) + \Delta_4(1) + 2\Delta_3(n)) = (U_1(1) + U_4(1) + 2U_3(n))n$;
7. $2V_3(n) = n(\Delta_1(1) - \Delta_4(1)) - (U_1(1) - U_4(1))n$.

Proof. Suppose that k-centralizing map Φ takes the following form

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) + \Delta_2(m) + \Delta_3(n) + \Delta_4(b) & T_1(a) + T_2(m) + T_3(n) + T_4(b) \\ V_1(a) + V_2(m) + V_3(n) + V_4(b) & U_1(a) + U_2(m) + U_3(n) + U_4(b) \end{bmatrix} \quad (1)$$

for all $\begin{bmatrix} a & m \\ n & b \end{bmatrix} \in \mathfrak{G}$ and $\Delta_1 : A \to A,\ \Delta_2 : M \to A,\ \Delta_3 : N \to A,\ \Delta_4 : B \to A$; $T_1 : A \to M,\ T_2 : M \to M,\ T_3 : N \to M,\ T_4 : B \to M$; $V_1 : A \to N,\ V_2 : M \to N,\ V_3 : N \to N,\ V_4 : B \to N$ and $U_1 : A \to B,\ U_2 : M \to B,\ U_3 : N \to B,\ U_4 : B \to B$ are $\mathfrak{R}$-linear maps. Since

$$[\Phi(G), G]_k \in \mathfrak{Z}(\mathfrak{G}) \quad \text{for all} \quad G \in \mathfrak{G}. \quad (2)$$

Now if we consider $G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ in (2), then it follows that

$$[\Phi(G), G]_k = \begin{bmatrix} 0 & (-1)^k T_1(1) \\ V_1(1) & 0 \end{bmatrix} \in \mathfrak{Z}(\mathfrak{G}).$$

This implies that $T_1(1) = 0 = V_1(1)$. Again on assuming $G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ in (2), we have $T_4(1) = 0 = V_4(1)$. On applying inductive approach with $G = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, we find that

$$[\Phi(G), G]_k = \begin{bmatrix} [\Delta_1(a), a]_k & (-1)^k a^k T_1(a) \\ V_1(a) a^k & 0 \end{bmatrix} \in \mathfrak{Z}(\mathfrak{G}).$$

This leads to $a^kT_1(a) = 0 = V_1(a)a^k$, $[\Delta_1(a), a]_k \in \mathfrak{Z}(A)_k$ and $0 \in \mathfrak{Z}(B)_k$. Also, it is easy to observe that $T_1(a) = T_1(a+1)$ and $V_1(a) = V_1(a+1)$. In view of Lemma 1, we arrive at $T_1(a) = 0 = V_1(a)$ for all $a \in A$. Further, we have $[\Delta_1(a), a]_k = 0$, i.e., Δ_1 is k-commuting map on A. Further, replacing a by $a+1$ in $[\Delta_1(a), a]_k = 0$, we conclude that $[\Delta_1(1), a]_k = 0$ for all $a \in A$ and hence $\Delta_1(1) \in \mathfrak{Z}(A)_k$.

On similar pattern for $G = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$, we can show that $T_4(b) = 0 = V_4(b)$ for all $b \in B$ and U_4 is k-commuting map on B and hence $U_4(1) \in \mathfrak{Z}(B)_k$. If $G = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ in (2), then it follows that

$$
\begin{aligned}
[\Phi(G), G]_k &= \begin{bmatrix} [\Delta_1(a) + \Delta_4(b), a]_k & 0 \\ 0 & [U_1(a) + U_4(b), b]_k \end{bmatrix} \\
&= \begin{bmatrix} [\Delta_1(a), a]_k + [\Delta_4(b), a]_k & 0 \\ 0 & [U_1(a), b]_k + [U_4(b), b]_k \end{bmatrix} \in \mathfrak{Z}(\mathfrak{G}).
\end{aligned} \tag{3}
$$

On using the fact Δ_1 and U_4 are k-commuting mappings on A and B respectively, we find that $[\Delta_4(b), a]_k \in \mathfrak{Z}(A)_k$ and $[U_1(a), b]_k \in \mathfrak{Z}(B)_k$ for all $a \in A$ and $b \in B$.

Suppose that $G = \begin{bmatrix} 1 & m \\ 0 & 0 \end{bmatrix}$ in (2) and consider

$$
[\Phi(G), G]_i = h_i = \begin{bmatrix} h_{i_{(11)}} & h_{i_{(12)}} \\ h_{i_{(21)}} & h_{i_{(22)}} \end{bmatrix} \quad \text{for all} \quad 0 \le i < k \quad \text{and} \quad h_k \in \mathfrak{Z}(\mathfrak{G}). \tag{4}
$$

This implies to

$$
\begin{aligned}
h_{i+1} &= \begin{bmatrix} h_{i+1_{(11)}} & h_{i+1_{(12)}} \\ h_{i+1_{(21)}} & h_{i+1_{(22)}} \end{bmatrix} \\
&= [h_i, G] \\
&= \left[\begin{bmatrix} h_{i_{(11)}} & h_{i_{(12)}} \\ h_{i_{(21)}} & h_{i_{(22)}} \end{bmatrix}, \begin{bmatrix} 1 & m \\ 0 & 0 \end{bmatrix} \right] \\
&= \begin{bmatrix} -mh_{i_{(11)}} & h_{i_{(11)}}m - mh_{i_{(22)}} - h_{i_{(12)}} \\ h_{i_{(21)}} & h_{i_{(21)}}m \end{bmatrix}.
\end{aligned}
$$

It follows that $h_{i+1_{(21)}} = h_{i_{(21)}}$ and hence $V_2(m) = h_{0_{(21)}} = h_{k_{(21)}}$. On using the fact $h_k \in \mathfrak{Z}(\mathfrak{G})$, we get $V_2(m) = 0$ for all $m \in M$. Therefore,

$$
h_0 = \begin{bmatrix} \Delta_1(1) + \Delta_2(m) & T_2(m) \\ 0 & U_1(1) + U_2(m) \end{bmatrix}
$$

and

$$h_1 = [h_0, G] = \begin{bmatrix} 0 & \Delta_1(1)m + \Delta_2(m)m - T_2(m) - mU_1(1) - mU_2(m) \\ 0 & 0 \end{bmatrix}.$$

Now by induction we arrive at $h_i = (-1)^{i-1}h_1,\ i > 0$ and hence $h_k = (-1)^{k-1}h_1$. This implies that $h_1 \in \mathfrak{Z}(\mathfrak{G})$. It follows that

$$T_2(m) = \Delta_1(1)m + \Delta_2(m)m - mU_1(1) - mU_2(m) \quad \text{for all} \quad m \in M.$$

On the similar pattern with $G = \begin{bmatrix} 0 & m \\ 0 & 1 \end{bmatrix}$, we find that

$$T_2(m) = mU_4(1) + mU_2(m) - \Delta_4(1)m - \Delta_2(m)m \quad \text{for all} \quad m \in M.$$

Combining the last two expressions, we arrive at

$$(\Delta_1(1) + \Delta_4(1) + 2\Delta_2(m))m = m(U_1(1) + U_4(1) + 2U_2(m))$$

and

$$2T_2(m) = (\Delta_1(1) - \Delta_4(1))m - m(U_1(1) - U_4(1)).$$

On assuming $G = \begin{bmatrix} 1 & 0 \\ n & 0 \end{bmatrix}$ and $G = \begin{bmatrix} 0 & 0 \\ n & 1 \end{bmatrix}$ respectively and applying similar techniques as above we can easily find that

$$V_3(n) = n\Delta_1(1) + n\Delta_3(n) - U_1(1)n - U_3(n)n \quad \text{for all} \quad n \in N.$$

and

$$V_3(n) = U_4(1)n + U_3(n)n - n\Delta_4(1) - n\Delta_3(n) \quad \text{for all} \quad n \in N.$$

The above two expressions leads to

$$n(\Delta_1(1) + \Delta_4(1) + 2\Delta_3(n)) = (U_1(1) + U_4(1) + 2U_3(n))n$$

and

$$2V_3(n) = n(\Delta_1(1) - \Delta_4(1)) - (U_1(1) - U_4(1))n.$$

Let us take $G = \begin{bmatrix} a & m \\ 0 & 0 \end{bmatrix}$ in (2) to find that $[\Delta_1(a), a]_k + [\Delta_2(m), a]_k \in \mathfrak{Z}(\mathfrak{G})$. Since Δ_1 is k-commuting map of A it follows that $\Delta_2(m) \in \mathfrak{Z}(A)_k$ by the arbitrariness of $m \in M$. In the similar way for $G = \begin{bmatrix} 0 & m \\ 0 & b \end{bmatrix}$ in (2), we have $U_2(m) \in \mathfrak{Z}(B)_k$ for all $m \in M$.

With the similar arguments as used above with $G = \begin{bmatrix} a & 0 \\ n & 0 \end{bmatrix}$ and $G = \begin{bmatrix} 0 & 0 \\ n & b \end{bmatrix}$ in (2) respectively, we observe that $\Delta_3(n) \in \mathfrak{Z}(A)_k$ and $U_3(n) \in \mathfrak{Z}(B)_k$ for all $n \in N$. □

As an immediate consequence of the above theorem, we obtain the following result:

Corollary 1 [12, Proposition 3.2] *Let $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ be a generalized matrix algebra over a commutative ring $\mathfrak{R}$. An $\mathfrak{R}$-linear map $\Phi : \mathfrak{G} \to \mathfrak{G}$ is a* k*-commuting map on $\mathfrak{G}$ if Φ has the following form*

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) + \Delta_2(m) + \Delta_3(n) + \Delta_4(b) & T_2(m) \\ V_3(n) & U_1(a) + U_2(m) + U_3(n) + U_4(b) \end{bmatrix}, \tag{5}$$

where $a \in A$; $b \in B$; $m \in M$; $n \in N$ and $\Delta_1 : A \to A$, $\Delta_2 : M \to \mathfrak{Z}(A)_k$, $\Delta_3 : N \to \mathfrak{Z}(A)_k$, $\Delta_4 : B \to \mathfrak{Z}(A)_k$, $T_2 : M \to M$, $V_3 : N \to N$, $U_1 : A \to \mathfrak{Z}(B)_k$, $U_2 : M \to \mathfrak{Z}(B)_k$, $U_3 : N \to \mathfrak{Z}(B)_k$, $U_4 : B \to B$ are $\mathfrak{R}$-linear maps satisfying the following conditions:

1. *Δ_1 is k-commuting map of A and $\Delta_1(1) \in \mathfrak{Z}(A)_k$;*
2. *U_4 is k-commuting map of B and $U_4(1) \in \mathfrak{Z}(B)_k$;*
3. *$(\Delta_1(1) + \Delta_4(1) + 2\Delta_2(m))m = m(U_1(1) + U_4(1) + 2U_2(m))$;*
4. *$2T_2(m) = (\Delta_1(1) - \Delta_4(1))m - m(U_1(1) - U_4(1))$;*
5. *$n(\Delta_1(1) + \Delta_4(1) + 2\Delta_3(n)) = (U_1(1) + U_4(1) + 2U_3(n))n$;*
6. *$2V_3(n) = n(\Delta_1(1) - \Delta_4(1)) - (U_1(1) - U_4(1))n$.*

In view of Lemma 3 and Theorem 1, it is easy to see that

Theorem 2 *Let $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ be a generalized matrix algebra over a commutative ring $\mathfrak{R}$ and $\Phi : \mathfrak{G} \to \mathfrak{G}$ be a* k*-centralizing map on $\mathfrak{G}$. If the following conditions are satisfied:*

1. *$\Delta_4(B) \subseteq \mathfrak{Z}(A)_k$ and $U_1(A) \subseteq \mathfrak{Z}(B)_k$;*

2. $\mathfrak{Z}(A)_k = \pi_A(\mathfrak{Z}(\mathfrak{G}))$ *and* $\mathfrak{Z}(B)_k = \pi_B(\mathfrak{Z}(\mathfrak{G}))$;

3. *there exist* $m_0 \in M,\ n_0 \in N$ *such that*

$$\mathfrak{Z}(\mathfrak{G}) = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle|\ a \in \mathfrak{Z}(A), b \in \mathfrak{Z}(B), am_0 = m_0 b, n_0 a = bn_0 \right\},$$

then Φ *is proper i.e.,* Φ *has the form* $\Phi = \lambda + \xi$, *where* $\lambda \in \mathfrak{Z}(\mathfrak{G})$ *and* $\xi : \mathfrak{G} \to \mathfrak{Z}(\mathfrak{G})$ *is an* $\mathfrak{R}$*-linear mapping.*

Also, we can see the implication of the above result in the settings of some nice examples of generalized matrix algebras (for detail see [12] and references therein) which follows directly:

Corollary 2 *Let* $\mathfrak{M}$ *be a von Neumann algebra without central summands of type* I_1. *Then any* k*-centralizing map on* $\mathfrak{M}$ *is proper.*

Corollary 3 [8, Theorem 1.1] *Let* $\mathfrak{A} = \mathrm{Tri}(A, M, B)$ *be a triangular algebra over a commutative ring* $\mathfrak{R}$ *and* $\Phi : \mathfrak{A} \to \mathfrak{A}$ *be a* k*-centralizing map on* $\mathfrak{A}$. *If the following conditions are satisfied:*

1. $\mathfrak{Z}(A)_k = \pi_A(\mathfrak{Z}(\mathfrak{A}))$ *and* $\mathfrak{Z}(B)_k = \pi_B(\mathfrak{Z}(\mathfrak{A}))$;

2. *there exist* $m_0 \in M,\ n_0 \in N$ *such that*

$$\mathfrak{Z}(\mathfrak{A}) = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle|\ a \in \mathfrak{Z}(A), b \in \mathfrak{Z}(B), am_0 = m_0 b \right\},$$

then Φ *is proper i.e.,* Φ *has the form* $\Phi = \lambda + \xi$, *where* $\lambda \in \mathfrak{Z}(\mathfrak{A})$ *and* $\xi : \mathfrak{A} \to \mathfrak{Z}(\mathfrak{A})$ *is an* $\mathfrak{R}$*-linear mapping.*

Now we describe the general form of k-skew centralizing maps on generalized matrix algebras as follows:

Theorem 3 *Let* $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ *be a* 2*-torsion free generalized matrix algebra over a commutative ring* $\mathfrak{R}$. *An* $\mathfrak{R}$*-linear map* $\Phi : \mathfrak{G} \to \mathfrak{G}$ *is a* k*-skew centralizing map on* $\mathfrak{G}$ *if* Φ *has the following form*

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) + \Delta_4(b) & T_2(m) \\ V_3(n) & U_1(a) + U_4(b) \end{bmatrix}, \tag{6}$$

where $a \in A;\ b \in B;\ m \in M;\ n \in N$ *and* $\Delta_1 : A \to A,\ \Delta_4 : B \to A,\ T_2 : M \to M,\ V_3 : N \to N,\ U_1 : A \to B\ ,\ U_4 : B \to B$ *are* $\mathfrak{R}$*-linear maps satisfying the following conditions:*

1. Δ_1 *is* k*-skew commuting map of* A;

2. U_4 *is* k*-skew commuting map of* B;

3. $\Delta_4(b) \circ_k a \in \mathfrak{Z}(A)_k$ *and* $U_1(a) \circ_k b \in \mathfrak{Z}(B)_k$;

4. $T_2(m) = -m\Delta_1(1)$ *and* $V_3(n) = -U_1(1)n$.

Proof. Assume that k-skew centralizing map Φ takes the following form

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) + \Delta_2(m) + \Delta_3(n) + \Delta_4(b) & T_1(a) + T_2(m) + T_3(n) + T_4(b) \\ V_1(a) + V_2(m) + V_3(n) + V_4(b) & U_1(a) + U_2(m) + U_3(n) + U_4(b) \end{bmatrix} \tag{7}$$

for all $\begin{bmatrix} a & m \\ n & b \end{bmatrix} \in \mathfrak{G}$ and $\Delta_1 : A \to A$, $\Delta_2 : M \to A$, $\Delta_3 : N \to A$, $\Delta_4 : B \to A$; $T_1 : A \to M$, $T_2 : M \to M$, $T_3 : N \to M$, $T_4 : B \to M$; $V_1 : A \to N$, $V_2 : M \to N$, $V_3 : N \to N$, $V_4 : B \to N$ and $U_1 : A \to B$, $U_2 : M \to B$, $U_3 : N \to B$, $U_4 : B \to B$ are $\mathfrak{R}$-linear maps. As we know that

$$\Phi(G) \circ_k G \in \mathfrak{Z}(\mathfrak{G}) \quad \text{for all} \quad G \in \mathfrak{G}. \tag{8}$$

Now if we assume $G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ in (8), then we find that

$$\Phi(G) \circ_k G = \begin{bmatrix} 2^k\Delta_1(1) & T_1(1) \\ V_1(1) & 0 \end{bmatrix} \in \mathfrak{Z}(\mathfrak{G}).$$

Therefore by using 2-torsion freeness, we get $\Delta_1(1) = T_1(1) = V_1(1) = 0$. Similarly with $G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, we find that $T_4(1) = V_4(1) = U_4(1) = 0$. Consider $G = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$ to get

$$\Phi(G) \circ_k G = \begin{bmatrix} \Delta_1(a) \circ_k a & a^kT_1(a) \\ V_1(a)a^k & 0 \end{bmatrix} \in \mathfrak{Z}(\mathfrak{G}).$$

This implies that $a^kT_1(a) = 0 = V_1(a)a^k$ and $\Delta_1(a) \circ_k a \in \mathfrak{Z}(A)_k$ & $0 \in \mathfrak{Z}(B)_k$. Also, it is easy to observe that $T_1(a) = T_1(a+1)$ and $V_1(a) = V_1(a+1)$. In view of Lemma 1, we arrive at $T_1(a) = 0 = V_1(a)$ for all $a \in A$. Also, we have $\Delta_1(a) \circ_k a = 0$, i.e., Δ_1 is k-skew commuting map on A.

Similarly for $G = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$, we find $T_4(b) = 0 = V_4(b)$ for all $b \in B$ and U_4 is k-skew commuting map on B. Replacing $G = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ in (8), we find that

$$\begin{aligned} \Phi(G) \circ_k G &= \begin{bmatrix} (\Delta_1(a) + \Delta_4(b)) \circ_k a & 0 \\ 0 & (U_1(a) + U_4(b)) \circ_k b \end{bmatrix} \\ &= \begin{bmatrix} \Delta_1(a) \circ_k a + \Delta_4(b) \circ_k a & 0 \\ 0 & U_1(a) \circ_k b + U_4(b) \circ_k b \end{bmatrix} \in \mathfrak{Z}(\mathfrak{G}). \end{aligned} \tag{9}$$

On using the fact Δ_1 and T_4 are k-skew commuting mappings on A and B respectively, we find that $\Delta_4(b) \circ_k a \in \mathfrak{Z}(A)_k$ and $U_1(a) \circ_k b \in \mathfrak{Z}(B)_k$ for all $a \in A$ and $b \in B$. Assume $G = \begin{bmatrix} 1 & m \\ 0 & 0 \end{bmatrix}$ in (8) and consider

$$\Phi(G) \circ_i G = h_i = \begin{bmatrix} h_{i_{(11)}} & h_{i_{(12)}} \\ h_{i_{(21)}} & h_{i_{(22)}} \end{bmatrix} \quad \text{for all} \quad 0 \le i < k \quad \text{and} \quad h_k \in \mathfrak{Z}(\mathfrak{G}). \tag{10}$$

Then

$$\begin{aligned} \begin{bmatrix} h_{i+1_{(11)}} & h_{i+1_{(12)}} \\ h_{i+1_{(21)}} & h_{i+1_{(22)}} \end{bmatrix} &= h_{i+1} = h_i \circ G \\ &= \begin{bmatrix} h_{i_{(11)}} & h_{i_{(12)}} \\ h_{i_{(21)}} & h_{i_{(22)}} \end{bmatrix} \circ \begin{bmatrix} 1 & m \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2h_{i_{(11)}} + mh_{i_{(11)}} & h_{i_{(11)}}m + mh_{i_{(22)}} + h_{i_{(12)}} \\ h_{i_{(21)}} & h_{i_{(21)}}m \end{bmatrix}. \end{aligned}$$

This implies that $h_{i+1_{(21)}} = h_{i_{(21)}}$ and hence $V_2(m) = h_{0_{(21)}} = h_{k_{(21)}}$. On using the fact $h_k \in \mathfrak{Z}(\mathfrak{G})$ we get $V_2(m) = 0$ for all $m \in M$. Therefore,

$$h_0 = \begin{bmatrix} \Delta_2(m) & T_2(m) \\ 0 & U_1(1) + U_2(m) \end{bmatrix}$$

and since $h_k \in \mathfrak{Z}(\mathfrak{G})$ and $\mathfrak{G}$ is 2-torsion free,

$$h_k = h_0 \circ_k G = \begin{bmatrix} 2^k\Delta_2(m) & h_{k_{(12)}} \\ 0 & 0 \end{bmatrix}.$$

Therefore, $\Delta_2(m) = 0$ for all $m \in M$. Also, we arrive at

$$h_0 = \begin{bmatrix} 0 & T_2(m) \\ 0 & U_1(1) + U_2(m) \end{bmatrix}$$

and hence

$$h_1 = h_0 \circ G = \begin{bmatrix} 0 & T_2(m) + mU_1(1) + mU_2(m) \\ 0 & 0 \end{bmatrix}.$$

By induction we have $h_i = h_1,\ i > 0$ and hence $h_k = h_1$. This implies that $h_1 \in \mathfrak{Z}(\mathfrak{G})$. It follows that

$$T_2(m) = -mU_1(1) - mU_2(m) \quad \text{for all} \quad m \in M.$$

On the similar pattern with $G = \begin{bmatrix} 0 & m \\ 0 & 1 \end{bmatrix}$, we find that $U_2(m) = 0$ for all $m \in M$. Combining last two expressions we arrive at $T_2(m) = -mU_1(1) \quad \text{for all} \quad m \in M$.

Let us take $G = \begin{bmatrix} 1 & 0 \\ n & 0 \end{bmatrix}$ and $G = \begin{bmatrix} 0 & 0 \\ n & 1 \end{bmatrix}$ respectively and applying similar techniques as above we can easily find that $T_3(n) = 0,\ \Delta_3(n) = 0,\ V_3(n) = -U_1(1)n - U_3(n)n$ and $U_3(n) = 0$ for all $n \in N$. These lead to $V_3(n) = -U_1(1)n \quad \text{for all} \quad n \in N$. □

Now we mention a significant result of this article as follows:

Theorem 4 *Let $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ be a 2-torsion free generalized matrix algebra over a commutative ring $\mathfrak{R}$. Then any k-semi centralizing derivation on $\mathfrak{G}$ is zero.*

Proof. Let Φ be a k-semi centralizing derivation on $\mathfrak{G}$. Then by Lemma 2, Φ has the following form

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) - mn_0 - m_0 n & am_0 + T_2(m) - m_0 b \\ n_0 a - bn_0 + V_3(n) & U_4(b) + nm_0 + n_0 m \end{bmatrix},$$

where $a \in A;\ b \in B;\ m, m_0 \in M;\ n, n_0 \in N$ and $\Delta_1 : A \to A,\ T_2 : M \to M,\ V_3 : N \to N,\ U_4 : B \to B$ are $\mathfrak{R}$-linear maps satisfying condition $(1) - (4)$ given in Lemma 2. Also from the proof of Theorem 1 or Theorem 3, it can be easily seen that $n_0 = V_1(1) = 0$ and $m_0 = T_1(1) = 0$. Now Φ takes the following form

$$\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \Delta_1(a) & T_2(m) \\ V_3(n) & U_4(b) \end{bmatrix}.$$

In view of k-centralizing case, condition (5) & (7) of Theorem 1 implies that $T_2(m) = 0$ and $V_3(n) = 0$ for all $m \in M$ and $n \in N$. Also, for k-skew centralizing case, we have $T_2(m) = 0$ and $V_3(n) = 0$ follows from condition (4) of Theorem 3.

Further, in view of condition (3) & (4) from Lemma 2 and using the faithfulness of M, for both k-centralizing and k-skew centralizing, we find that $\Delta_1(a) = 0$ and $U_4(b) = 0$ for all $a \in A$ and $b \in B$. Thus we conclude that $\Phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ for all $\begin{bmatrix} a & m \\ n & b \end{bmatrix} \in \mathfrak{G}$. □

In view of the above theorem, we get the following results:

Corollary 4 *Let $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ be a 2-torsion free generalized matrix algebra over a commutative ring $\mathfrak{R}$. Then any* k*-semi commuting derivation on $\mathfrak{G}$ is zero.*

Corollary 5 *Let $\mathfrak{M}$ be a von Neumann algebra without central summands of type I_1. Then any* k*-semi centralizing (commuting) derivation on $\mathfrak{M}$ is zero.*

Corollary 6 *Let $\mathfrak{A} = \mathrm{Tri}(A, M, B)$ be a 2-torsion free triangular algebra over a commutative ring $\mathfrak{R}$. Then any* k*-semi centralizing (commuting) derivation on $\mathfrak{A}$ is zero.*

4 For future discussions

In view of [4, Propostion 2.1, 2.2], we can write the structure of automorphisms on generalized matrix algebras respectively as follows:

Lemma 4 *Let $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ be a generalized matrix algebra and $(\gamma, \delta, \mu, \nu, m_0, n_0)$ be a 6-tuple such that $\gamma : A \to A$ and $\delta : B \to B$ are algebraic automorphisms, $\mu : M \to M$ is $\gamma - \delta -$bimodule automorphism, $\nu : N \to N$ is a $\delta - \gamma -$bimodule automorphism and $m_0 \in M$ & $n_0 \in N$ are fixed elements such that following conditions are satisfied:*

(i) $[m_0, N] = 0$ *and* $(N, m_0) = 0$,

(ii) $[M, n_0] = 0$ *and* $(n_0, M) = 0$,

(iii) $[\mu(m), \nu(n)] = \gamma([m.n])$ *and* $(\nu(n), \mu(m)) = \delta((n, m))$.

Then the map $\phi : \mathfrak{G} \to \mathfrak{G}$ defined by

$$\phi\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \gamma(a) & \gamma(a)m_0 - m_0\delta(b) + \mu(m) \\ n_0\gamma(a) - \delta(b)n_0 + \nu(n) & \delta(b) \end{bmatrix}$$

is an algebraic automorphism.

Lemma 5 *Let* $\mathfrak{G} = \mathfrak{G}(A, M, N, B)$ *be a generalized matrix algebra and* $(\rho, \sigma, \mu, \nu, m_*, n_*)$ *be a 6-tuple such that* $\rho : A \to B$ *&* $\sigma : B \to A$ *are algebraic automorphisms,* $\mu : (M, +) \to (N, +)$ *&* $\nu : (N, +) \to (M, +)$ *are group automorphisms such that* $\mu(amb) = \rho(a)\mu(m)\sigma(b)$ *&* $\nu(bna) = \sigma(b)\nu(n)\rho(a)$ *for all* $a \in A, b \in B, m \in M, n \in N$ *and* $m_* \in M$ *&* $n_* \in N$ *are fixed elements such that following conditions are satisfied:*

(i) $[m_*, N] = 0$ *and* $(N, m_*) = 0$,

(ii) $[M, n_*] = 0$ *and* $(n_*, M) = 0$,

(iii) $(\mu(m), \nu(n)) = \rho([m, n])$ *and* $[\nu(n), \mu(m)] = \sigma((n, m))$.

Then the map $\psi : \mathfrak{G} \to \mathfrak{G}$ *defined by*

$$\sigma\left(\begin{bmatrix} a & m \\ n & b \end{bmatrix}\right) = \begin{bmatrix} \sigma(a) & m_*\rho(a) - \sigma(b)m_* + \nu(n) \\ \rho(a)n_* - n_*\sigma(b) + \mu(m) & \rho(b) \end{bmatrix}$$

is an algebraic automorphism.

Now at this point, it is natural to raise a question:

Question 5 *What is the most general form of* k*-semi centralizing (commuting) automorphisms on generalized matrix algebras and which constraints are needed to apply on generalized matrix algebras?*

5 Conclusions

In this article, we find out the structures of k-centralizing and k-skew centralizing maps on generalized matrix algebras. Further, we conclude that k-centralizing map has proper form. In addition, we prove that k-semi centralizing derivation is zero on generalized matrix algebras. In the end of article, we draw the attention of readers towards the investigation of k-semi centralizing (commuting) automorphisms on generalized matrix algebras for future research works.

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