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Asymmetry in Okun's Law Revisited: New Evidence on the Cyclical Unemployment–Cyclical Output Nexus in the Free State Province (South Africa) Using a Nonlinear ARDL Model

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Abstract: This paper examines the asymmetric unemployment–output nexus employing the nonlinear autoregressive distributed lag (NARDL) model. Cyclical components of unemployment and real output are estimated from annual data covering 1994–2019, using the Hodrick–Prescott and the Corbae–Ouliaris detrending techniques. Controlling for structural change effects, we find a statistically significant asymmetric cyclical unemployment–cyclical output relationship in the long run and the short run in the Free State province (South Africa), regardless of the detrending method used. Specifically, empirical results show that a one-percent increase in cyclical output can reduce cyclical unemployment between 0.70 and 0.87 percentage points, albeit conditioned on sustained economic growth. Also, the significant long-run coefficients of cyclical output reveal that an economic upturn between 1.88 and 2.03 percent would reduce unemployment by one percent. Based on our findings, proactive implementation of macro-fiscal policies consisting of demand-and-supply-side interventions is required to spur economic growth and lower the prevailing high unemployment rate in the province.

Keywords: Okun's law, NARDL, Hodrick–Prescott filter, Corbae–Ouliaris filter, South Africa

JEL Classification: C51, E24, E32, J64

1. Introduction

The seminal work by Okun (1962) formalized the widely accepted notion of the inverse relationship between output and unemployment, connecting activity in the goods market to activity in the labour market. This relationship, universally known as Okun's law, laid an enduring theoretical foundation and extensively researched theory in macroeconomics (see e.g. Ball et al., 2017; Weber, 1995; Prachowny, 1993; Gordon, 1984). Broadly, Okun (1962) found that a three-percentage-point increase in cyclical output leads to a one-percentage-point decrease in cyclical unemployment in the United States (US), producing an empirical regularity of 3:1 ratio encapsulating the output–unemployment trade-off generally regarded as a useful "rule of thumb", providing policymakers with a benchmark to measure the cost of higher unemployment (see Palley, 1993: 144; Moosa, 1997: 664; Lee, 2000: 331).[1]

However, as the empirical literature assessing Okun's law develops, three shortcomings are evident. Firstly, a large number of studies confirming Okun's law are devoted to developed economies, most notably the US and European countries (see e.g. Ball et al. 2017; Huang and Yeh, 2013; Holmes and Silverstone, 2006; Altissimo and Violante, 2001), but developing economies are less studied, particularly African countries where high unemployment and anaemic economic growth are rampant.[2]

Secondly, numerous extant studies limit their analysis validating Okun's law to the country (or aggregate) level solely utilizing national data, ignoring the importance of similar investigation using regional (i.e. provincial or state) data (see Freeman, 2000: 558; Christopolous, 2004: 612; Villaverde and Maza, 2009: 290; Binet and Facchini, 2013: 2). Thirdly, several studies modelled the output–unemployment relationship solely based on the linear assumption initially posited by Okun's law. This model specification is not only biased since it ignores the possibility of an asymmetric link between these macro-variables (Palley, 1993: 145; Shin et al., 2014: 299) but also produces forecasting errors and spurious or inconclusive inferences, resulting in flawed prescriptions that render formulated policy ineffective (see Harris and Silverstone, 2001; Virén, 2001; Silvapulle et al., 2004).

The attempt to remedy the deficiencies in the extant studies has led to the emergence of two nascent strands of empirical literature. On the one hand, studies in the first strand of the emergent literature provide irrefutable evidence of asymmetry in Okun's law (see Palley, 1993; Lee, 2000; Harris and Silverstone, 2001; Virén, 2001; Silvapulle et al., 2004; Cuaresma, 2003, Shin et al., 2014 among others), invalidating the original linear assumption typically used in modelling output–unemployment relationship in many studies. Particularly the initial assumption that output and unemployment are linearly related as postulated in Okun's law implies that both positive output (economic upturns) and negative output (downturns) have similar absolute effects on unemployment (Shin et al., 2014: 229); however, due to structural

1 For example, see Moosa (1997) for empirical proof affirming Okun's coefficient as the most suitable benchmark for measuring unemployment cost.

2 See e.g. Ibourk and Elaynaoui (2024) for a recent survey of studies validating Okun's law in African countries.

change effects and/or the cyclical nature of the business cycle, a relative change in cyclical unemployment would depend on whether the cyclical output is negative (economic upswing) or positive (downswing), ascertaining the presence of an asymmetric relationship between unemployment and changes in output growth (see Neftçi, 1984; Palley, 1993: 145; Cuaresma, 2003: 440).

On the other hand, studies in the second strand of the emergent literature extend the analysis of the output–unemployment relationship to the regional (i.e. sub-country, provincial, or state) level, asserting that the analysis of the output–unemployment relationship within the context of Okun's law using only national data suffers from aggregation bias, masking the existing interregional disparity (i.e. differences or heterogeneity), which can be significant, as well as the variation in estimates of Okun's coefficient at the regional level due to region-specific factors such as structural change effects, dissimilar economic structures, different levels of population, unemployment, output growth, and industrialization (see e.g. Adanu, 2005; Binet and Facchini, 2013; Durech et al., 2014).

Surprisingly, studies using regional (i.e. provincial- or state-level) data have documented strong evidence confirming that Okun's law holds in some regions but is invalid in other regions within the same country, and they estimated that Okun's coefficients at a country (i.e. aggregate or national) level differ from estimates of Okun coefficients at the regional level (or across the regions) where Okun's law holds (see e.g. Guisinger et al. (2018) for the US, Villaverde and Maza (2009) for Spain, and Freeman (2000) for the US).[3]

Indisputably, assessing the output–unemployment relationship at the regional level is important for empirical and policy-related reasons. Empirically, such analysis provides useful information on the magnitude (or size) of Okun's coefficients across regions and the influence of regional differences in the responsiveness of output to reductions in unemployment (Freeman, 2000: 558). In policymaking, evidence from regional analysis of Okun's law could aid policymakers in adopting effective stabilization policies to deal with unemployment at both the regional and the national levels. As such, efficient demand management policies could be implemented by policymakers to lower unemployment in regions where Okun's law holds, while alternative strategies (for example, raising infrastructure spending or providing tax incentives to attract foreign investment or monetary incentives to firms to train unskilled job seekers) can be used in regions where Okun's law is found invalid (Christopoulous, 2004: 612).

Hitherto, only a few studies have tested asymmetric Okun's law in South Africa that rely mostly on national data, thus restricting their analysis to the country level, except Kavase and Phiri (2020), who examined asymmetric Okun's relationship at the provincial level, in South Africa. To this end, very little is known about the nature (i.e. symmetric or asymmetric) of the existing linkages between changes in

3 Refer to the literature review section for reported findings from related regional studies.

unemployment and changes in output at the provincial level in South Africa. This deficiency makes effective policymaking elusive at the national and provincial levels, rendering nationally adopted policies based on the recommendations from studies using national data ineffective.

This present paper fills the gap in the extant literature by examining the existence of asymmetric Okun's law in the Free State province in South Africa utilizing provincial data. Our analysis is relevant for two main reasons: firstly, analysed available data from Statistics South Africa (Stats SA, 2019a, 2020) reveal that the Free State province is among the top three regions experiencing a persistently high rate of unemployment, which is above 25% (but grew to 38% in 2018), and anaemic economic growth, averaging about 2% between 2000 and 2019 (see *Figure A1* in the *Appendix*). Even during periods of relative economic expansion, the provincial unemployment rate remains elevated. For example, between 1997 and 2007, the regional real gross domestic product (GDPR) for the Free State province rose to 3.7% from around 1%; in contrast, the unemployment rate surged to 28% from 25% during the same period. Subsequently, the real GDPR growth dramatically declined to around 1.4% in 2017 before turning negative (-0.2%) in the following year, while the provincial unemployment rate surged to 33.7% in 2017 and rose further to nearly about 38% in 2018. Observably, periods of relative expansion (i.e. 2001–2005 and 2010–2012) in the Free State provincial economy do not reduce unemployment, contradicting Okun's law, but align with the debatable "jobless growth" phenomenon in South Africa (see e.g. Phiri, 2014; Leshoro, 2013; Hodge, 2009; Marinkov and Geldenhuys, 2007; Casale et al., 2004). Hence, evidence validating the existence of asymmetric Okun's law in the focused region using provincial data might explain why unemployment is typically irresponsive to output growth shock, especially during expansion.

Secondly, the South African government has ratified at least six macro-policies since 1994, which have failed in achieving the common goal of raising annual economic growth above 5% and reducing the unemployment rate below 14% (see Stats SA, 2019b; NDP, 2012; NGP, 2010; The Presidency, 2006, 2007),[4] as the country grapples with a stubbornly high unemployment rate (on average, above 24%) and weak economic growth, which only peaked at around 5% in the 2000s (i.e. from 2005 to 2007, and steadily fell below 1% in 2018) (see *Figure A2* in the *Appendix*). In this regard, evidence of an asymmetric unemployment–output relationship in the Free State province could also explain why the nationally adopted policies are ineffective in reducing unemployment and spurring output

4 The nationally adopted policies include *the Reconstruction and Development Programme* (RDP) in 1994, *the Growth, Employment, and Redistribution Strategy* (GEAR) in 1998, *the Accelerated and Shared Growth Initiative for South Africa* (ASGISA) in 2004, *the Joint Initiative for Priority Skills Acquisition* (JIPSA) in 2007, *the New Growth Path* (NGP) in 2010, and recently *the National Development Plan* (NDP) in 2012. Out of these, only the ASGISA produced an average annual growth of about 5% between 2005 and 2007.

growth (Virén, 2000: 253). All in all, evidence from our analysis supporting the existence of asymmetric Okun's relationship in the Free State province could be useful in determining the desirable or optimal output growth required to reduce unemployment, which in turn enhances the effectiveness of policymaking (Harrison and Silverstone, 2001; Silvapulle et al., 2004; Moosa, 2008).

To this end, this paper tests asymmetry in Okun's law in the Free State province by examining the relationship between cyclical unemployment and cyclical output over the period from 1996 to 2019, employing the popular nonlinear autoregressive distributive lag (NARDL) model developed by Shin et al. (2014). This model is the most apt for our analysis because it is superior to many advanced econometric tools given its ability to decompose cyclical output variables into partial sum processes of positive and negative changes capturing the responsiveness to cyclical unemployment, to positive and negative cyclical outputs, flexibly captures both long- and short-run asymmetries existing between variables, proficiently deals with misspecification bias, and produces robust inferences when sample size is small.[5]

To the best of our knowledge, only the study by Kavase and Phiri (2020) has examined asymmetric Okun's law across South African provinces and has found that Okun's law does not hold in the Free State province, but a positive long-run relationship between cyclical unemployment and cyclical output exists, which contradicts Okun's law. Even so, this previous study suffers from two shortcomings: firstly, the authors ignore the impact of structural change on the asymmetric relationship between cyclical unemployment and cyclical output, which in turn could affect the stability of estimated Okun's coefficients (see e.g. Lee, 2000: 348; Sögner and Stiassny, 2002), particularly at the provincial level (Apergis and Rezitis, 2003:114). Secondly, in this previous study, only one detrending method is used to generate cyclical variables; nonetheless, this procedure is inadequate to confirm the robustness of variation in estimated Okun's coefficients across the provinces, which may differ when two or more detrending methods are utilized (see Freeman, 2000; Adanu, 2005; Villaverde and Maza, 2009; Phiri, 2014; Marinkov and Geldenhuys, 2007, among others).

We remedied limitations in the previous study closest to ours, in three ways. Firstly, unlike Kavase and Phiri (2020), we applied the Hodrick–Prescott (HP, 1997) and the Corbae–Oularis (CO, 2006) detrending methods to estimate cyclical variables used in constructing two separate asymmetric models, allowing us to control the sensitivity of Okun's coefficients in the presence of long- and short-run asymmetries (Lee, 2000: 346) and also to affirm the robustness of estimated Okun's coefficients, which can be obscured when only one detrending method is used. Secondly, the structural change effect on estimates of Okun's coefficients is accounted for, by adding dummy variables into our asymmetric models. Thirdly,

5 The NARDL model is an extension of the ARDL model introduced by Pesaran et al. (2001), and hence it has similar advantages.

the reliability of the obtained results is assessed by performing a sensitivity analysis, which entails re-estimating the baseline models with cyclical variables of real GDP per capita (instead of the real cyclical output initially used) to capture the influence of population growth on the validity of Okun's law (Adanu, 2005) and on the asymmetric links between the cyclical output and unemployment variables in the FS province. On this basis, our results can be considered reliable, robust, and useful for policy formulation, hence our significant contribution to the literature.

The rest of this paper is structured as follows. Section 2 discusses relevant theory and surveys empirical studies in two emergent strands of the literature assessing the validity of Okun's law. Section 3 presents the data and detrending methods used to generate cyclical variables. Modelling the gap specification of Okun's law employing the NARDL modelling framework is provided in Section 4, and empirical results are discussed in Section 5. Section 6 concludes with some policy recommendations.

2. Literature Review

This sub-section surveys relevant studies in two nascent strands of the vast literature that have made significant contributions which have refined the empirical analysis of Okun's law, with studies in the first strand of the literature assessing asymmetry in Okun's law and those in the second strand of literature extending the analysis of Okun's law to the regional (i.e. provincial or state) level.

2.1. Theoretical Discussion

The importance of Okun's law has long been established from both theoretical and empirical perspectives. Theoretically, the combination of Okun's law with the Phillips curve produces an aggregate supply curve for an economy (Prachowny, 1993: 331), creating a link between the inflation rate, unemployment rate, and economic growth (Marinkov and Geldenhuys, 2007: 374). This linkage is important for macroeconomic policy, particularly in determining the desirable or optimal growth rate, and as a prescription for reducing unemployment (Silvapulle et al. 2004: 354; Moosa, 2008: 8). Empirically, the Okun coefficient, which measures the responsiveness of unemployment to changes in output growth, is a useful "rule of thumb" in forecasting and policymaking (Harris and Silverstone, 2001: 1; Moosa, 2008: 8).

Nevertheless, Neftçi (1984) observed unemployment exhibiting a differentiated response during periods of economic expansion and contraction in the US, which reinforces the notion that an asymmetric link could exist between changes in unemployment and changes in output, during different phases of the business cycle. As such, the response of unemployment to a positive cyclical output shock (i.e. economic expansion) may differ (in magnitude) from a negative cyclical output shock (i.e. economic contraction).

In theory, unemployment could respond asymmetrically to changes in output during the business cycle, at least in three ways. Firstly, if the cost of reducing the quantity of labour (e.g. retrenchment packages) is perceived to be high during the economic downturn, firms may adopt a "labour hoarding" strategy to retain existing workers but reduce their working hours or the number of shifts to maximize profit or lower total costs, and they can adopt a "labour spreading" strategy when the economy recovers or expands by extending employees' working hours through overtime or increasing the number of shifts, without hiring additional workers (Palley 1993: 149). Secondly, based on the notion of labour market hysteresis, cyclical output shocks could exert a permanent effect on structural unemployment, to the extent that unemployment does not revert to its pre-crisis level after economic recovery (Blanchard and Summers, 1986). Thirdly, firms may not hire the same quantity of labour shed to optimize revenue or cope with high operational costs during economic downturns when the economy recovers, given equal magnitude of negative and positive shocks (Shin et al. 2014: 299).

Indeed, both Harrison and Silverstone (2001: 1) and Silvapulle et al. (2004: 356) have stressed the importance of testing for asymmetry in Okun's law on the following grounds: theoretically, it is useful for discriminating between competing theories of joint labour and goods market behaviour. In policymaking, the knowledge about the extent of asymmetry in the output–unemployment relationship is useful in formulating structural (e.g. labour market reforms) and stabilization policies (e.g. suitable monetary policy stance). Empirically, ignoring asymmetry in Okun's law, when it exists, generally leads to model misspecification, poor forecasting and spurious inferences in hypothesis testing by rejecting the null hypothesis that both output and unemployment are asymmetrically cointegrated, that is, a long-run relationship exists between the two macro-variables. Consequently, our analytical strategy in this study is guided by the aforementioned theoretical reasoning.

2.2. Empirical Evidence

Among the studies investigating asymmetric Okun's relationship in the US, the earliest contribution by Neftçi (1984) shows that the unemployment rate has gone through much sharper increases when the US economy contracts than when it declines during expansions, while Rothman (1991) found an asymmetric response in unemployment to positive and negative output growth shocks, and Palley (1993) reported similar findings – however, he finds a stronger response in unemployment to negative output growth using a dynamic model.

Subsequently, Altissimo and Violante (2001) also find evidence for an asymmetric unemployment–output relationship owing to the larger impact of propagated shocks on unemployment than output during recessions in the US, in a nonlinear VAR model. Meanwhile, Cuaresma (2003) employed a regime-dependent specification of Okun's law and found a more significant asymmetric contemporaneous effect of cyclical output on cyclical unemployment during economic recessions than during expansions. Using a dynamic structural time-series model, Silvapulle et al. (2004) discovered that an inverse asymmetric cyclical output–cyclical unemployment relationship exists in the US but that the contemporaneous effect of positive cyclical output on cyclical unemployment quantitatively differs from negative ones. Alike, Holmes and Silverstone (2006) used a Markov regime-switching model to capture asymmetries within and across regimes and found a significant asymmetric inverse relationship between cyclical output and unemployment in the U.S., especially during economic expansion.

Considering studies investigating the validity of asymmetric Okun's law in European countries belonging to the Organisation for Economic Co-operation and Development (OECD), for example, Lee (2000) examined asymmetric Okun's relationship in 16 OECD countries using a static model which allows changes (i.e. negative and positive) in unemployment to determine output growth and found a significantly higher Okun coefficient for decreases (as compared to increases) in the unemployment rates for Finland, Japan, and the US, but the opposite holds for Canada, France, and the Netherlands.

Virén (2001) conducted a similar analysis in 20 OECD countries relying on an asymmetric error correction-based model (in which changes in unemployment are determined by positive and negative changes in output) and obtained results confirming asymmetric Okun's relationship with output growth having a strong effect on unemployment during periods of low unemployment and higher output (and vice versa). Likewise, Harris and Silverstone (2001) also estimated an asymmetric error correction model (where Okun's coefficient is either above or below long-run equilibrium) consisting of 7 OECD countries, and they found unemployment reacting asymmetrically to contemporaneous changes in output contingent on upswings or downswings in the business cycle, validating asymmetric Okun's law for Australia, Japan, New Zealand, United Kingdom, United States, and Germany.

In the same vein, Huang and Yeh (2013) found highly significant asymmetric unemployment–output trade-offs for 53 OECD countries both in the short and long run at the state and country level, in a panel ARDL model, while Shin et al. (2014) confirmed the existence of a negative asymmetric relationship between cyclical output and unemployment in the US, Canada, and Japan utilizing a NARDL model, and Tang and Bethencourt (2017) found evidence for long-run and short-run asymmetries between cyclical unemployment and cyclical output in most of the 17 Eurozone countries studied, with the help of a NARDL model.

So far, few studies have examined the existence of asymmetric Okun's law in South Africa. Among these, in an asymmetric error-correction model accounting for structural breaks, Marinkov and Geldenhuys (2007) found an asymmetric cyclical output–cyclical unemployment relationship existing in South Africa and highly significant asymmetric contemporaneous effects of cyclical output on cyclical unemployment, with estimates of Okun coefficients ranging from -0.77 to -0.18 across the different detrending methods used. Also, Phiri (2014) reported similar findings on asymmetric Okun's law in a dataset covering the period of 2000–2013; however, further results reveal that unemployment granger causes economic growth in the long run, supporting the notion of "jobless growth phenomenon" prevailing in South Africa.

More recently, computing a NARDL model with quarterly data spanning the period from 1994Q1 to 2017Q4, Mazorodze and Siddiq (2018) found a significant asymmetric cyclical unemployment–cyclical output relationship in the long run but a symmetric relationship in the short run, partly due to dominant unionized labour market in South Africa. The authors conclude that a 10% increase in positive (negative) cyclical output reduces (raises) unemployment by 8% in the long run. In contrast, Sere et al. (2020) found an insignificant positive asymmetric relationship between changes in output on unemployment in the long run and a statistically significant inverse relationship between these variables in the short run, in an estimated NARDL model over the period from 1994Q1 to 2019Q4.

Considering studies in the second strand of literature assessing the validity of Okun's law at the sub-country level using regional (i.e. provincial or state level) data, findings from the earliest study by Freeman (2000) validate output–unemployment relationship consistent with Okun's law in 8 regional economies in the US over the period of 1958–1998 and evidence of trivial interregional differences at the regional level, with varying Okun's coefficients ranging between 1.87 and 3.57, in contrast to the coefficient of about 2 found for the US economy, at the national level.

Elsewhere, Apergis and Rezitis (2003) found that Okun's law holds in 8 regional economies studied in Greece over the period from 1960 to 1997, with Okun's coefficients ranging from 1.15 to 3.56 across the regions, and insignificant interregional disparity irrespective of the detrending methods used, except in the cases of Epirus (2.97 to 3.19) and North Aegean (3.56 to 3.69) Islands that exhibit higher Okun's coefficients attributed to structural change. In a subsequent study, Christopoulos (2004) found that Okun's law only held in 6 out of 13 regional Greek economies investigated over the period of 1971–1993 and that regional Okun's coefficients varied between 0.37 and 1.70.

Similarly, Adanu (2005) considered the existence of Okun's law in 10 Canadian provinces using regional data covering the period of 1981–2001 and discovered a significant regional difference with weighted-provincial Okun's coefficients ranging between 1.32 and 1.58 across the two detrending methods employed. Also, Villaverde and Maza (2009) found that Okun's law applies to 15 out of 17

Spanish regional economies studied during the period of 1980–2004 and variation in Okun's coefficients with values ranging from 0.32 to 1.55 across the regions, compared to the coefficient of 0.91 for the entire Spain. Alike, Binet and Facchini (2013) confirmed Okun's law in 14 out of 22 regional economies in France, and varying Okun's coefficients across the regions, with values between 0.91 and 1.81.

More recently, Durech et al. (2014) assessed regional disparity in the responsiveness of output to changes in unemployment across 14 regions in the Czech Republic and 7 regions in Slovakia. Their results validate Okun's law in 11 (out of 14) regions in Czechia and 5 (out of 8 regions) in Slovakia, with strong evidence of significant regional disparity, and regional Okun's coefficients ranging from 2.58 (for Bratislava) to 3 (for Prague) in Czechia and from 0.92 (for Olomouc) to 1.11 (for Nitra) in Slovakia. Also, the empirical results of Guisinger et al. (2018) show that Okun's law holds in 47 out of 51 states examined in the US over the period from 1977 to 2012 and found significant regional disparity (i.e. estimated Okun's coefficients of about 21.6% of state pairs are statistically different) with variations in Okun's coefficients across the states, ranging from 1.25 (for Colorado) to 4.38 (for North Dakota), compared to the coefficient of about 2.03 obtained for the US at the national (aggregate) level.

Focusing on South Africa, Kavase and Phiri (2020) discovered that Okun's law applied to only 2 out of 9 provinces using provincial data over the period of 1996–2016 and weakly significant Okun coefficients with values varying from 0.51 (for Western Cape) to 1.91 (for KwaZulu-Natal), while there is also evidence of a significant asymmetric relationship between cyclical output and cyclical unemployment in both the long and the short run existing in these provinces, but other provinces mostly exhibit significant short-run asymmetries, except the Free State province, where a positive long-run asymmetry is detected, which contradicts Okun's law.

Aside from Kavase and Phiri (2020), empirical investigation on asymmetric Okun's law at the provincial (regional) level in South Africa is inexistent. Since our work remedies the shortcomings in this earlier study, as previously mentioned, our result can be considered more reliable, robust, and useful in policy formulation. Based on this, our contribution to the extant literature is significant and timely.

3. Data and Treatment

Our estimated dynamic models contain annual time series on the gross domestic product (GDP at constant prices, 2010 = 100), GDP per capita (constant prices, 2010 = 100), and the total unemployment for the Free State province, from 1999 to 2019. Nominal series (i.e. GDP and GDP per capita) are converted to real series using the Consumer Price Index (CPI, 2010 = 100). Annual data are primarily sourced from the Statistics South Africa (https://www.statssa.gov.za/), and S&P Global South Africa Regional eXplorer (https://www.ihsmarkit.co.za/) databases. Data are seasonally adjusted utilizing the Seasonal and Trend decomposition (STL) method (see Cleveland et al., 1990).[6]

Generally, the gap specification of Okun's law requires unobserved information about unemployment and output trends. Since there is no definite guideline in the empirical literature on which detrending method (i.e. filters) should be used to generate these trend series, we estimated cyclical (or gap) and trend components of the real GDP and unemployment growth rates from their respective observed series, y_t and u_t applying the Hodrick–Prescott (1997) and Corbae–Ouliaris (2006) filtering methods.

Characteristically, the HP filter is a two-sided linear that computes the smoothed series y^* of y by minimizing the variance of y around y^* subject to a penalty function that contains the change in the trend growth of y^*, written in the form as follows:

$$\Theta = \sum_{t=1}^{T} (y_t - y_t^*)^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^* - y_t^*) - (y_t^* - y_{t+1}^*)]^2 \tag{1}$$

In Eq. (1), the penalty parameter λ, controls the smoothness of the series y^* with the residual $y_t - y^*$ indicating the deviation from the trend and is commonly referred to as the business cycle component.

In addition to the HP filter, as previously stated, cyclical components (or gap variables) of the selected time series are also generated utilizing the CO filter (a frequency-domain filter), which is superior in extracting cyclical components without losing observations at the endpoints of the series. As such, the CO filter generates detrended series with super-consistent finite sample properties that are statistically reliable and asymptotically converge to their true growth cycle (Corbae and Ouliaris, 2006). In contrast to time-domain filters (for example, HP-, Band-pass-, and Baxter King-filters) that suffer from end-point problems, the CO filter is unsusceptible to end-point problems.

More specifically, by using Monte Carlo simulations, Corbae and Ouliaris (2006) show that the CO filter has superior finite sample properties by producing a detrended series that is statistically reliable (with much lower mean-squared error) and asymptotically converges to their true growth cycle. For a brief exposition, consider a non-stationary time series x_t with a deterministic component z_t and an unobserved stochastic component $\widetilde{x}_t$ denoted as:

6 The STL decomposes annual, quarterly, and monthly data into seasonal and trend components, making it a versatile and superior decomposition method compared to the commonly used ARIMA-based decomposition methods (e.g. TRAMO-SEATs and X-11). This decomposition method generates robust seasonal series using Loess estimation, producing robust estimates of the trend-cycle and seasonal components from time-series data with irregular patterns and/or missing values.

$$x_t = \Pi_2^{'} z_i + \tilde{x}_t \,, \tag{2}$$

with z_t being a $p + 1$ dimensional deterministic sequence and $\tilde{x}_t$ a zero-mean time series. Given that $\tilde{x}_t$ is a first-differenced [$I(1)$] stationary series defined as $\Delta x_t = v_t$ with a Wold representation:

$$v_t = \Sigma_{j=0}^{\infty} cj \xi_{t-j} \,, \tag{3}$$

where spectral density $f_{vv}(\lambda) > 0$ for all λ, coefficients $c_j = \Sigma_{j=0}^{\infty} j^{\frac{1}{2}} \left| c_j \right| < \infty$, and $\xi = iid(0, \sigma^2)$, which has finite fourth moments. Here, the discrete Fourier transform of $\tilde{x}_t$ for $\lambda_s \neq 0$ is given by:

$$w_{\tilde{x}}(\lambda_s) = \frac{1}{1 - e^{i\lambda_s}} w_v(\lambda_s) - \frac{e^{i\lambda_s}}{1 - e^{i\lambda_s}} \frac{[\tilde{x}_n - \tilde{x}_0]}{n^{1/2}}, \tag{4}$$

where $\lambda_s = \dfrac{2\pi s}{n} (s = 0, 1, \ldots, n-1)$ are the fundamental frequencies of a sample n.

But the second term in Eq. (4) shows that the Fourier transform that follows $I(1)$ process is not asymptotically independent across fundamental frequencies because it is a deterministic trend in the frequency domain with a random coefficient $\dfrac{[\tilde{x}_n - \tilde{x}_0]}{n^{1/2}}$, rather they are frequency-wise dependent by virtue of the component $n^{1/2} \tilde{x}_n$, which produces a common leakage to all frequencies, $\lambda_t \neq 0$, and in the limit as $n \to \infty$. This leakage problem and bias estimates of cyclical components are fixed by de-trending the second term in Eq. (4) using a frequency-domain regression to generate an unbiased estimate of $w_{\tilde{x}}(\lambda_s)$ represented as:

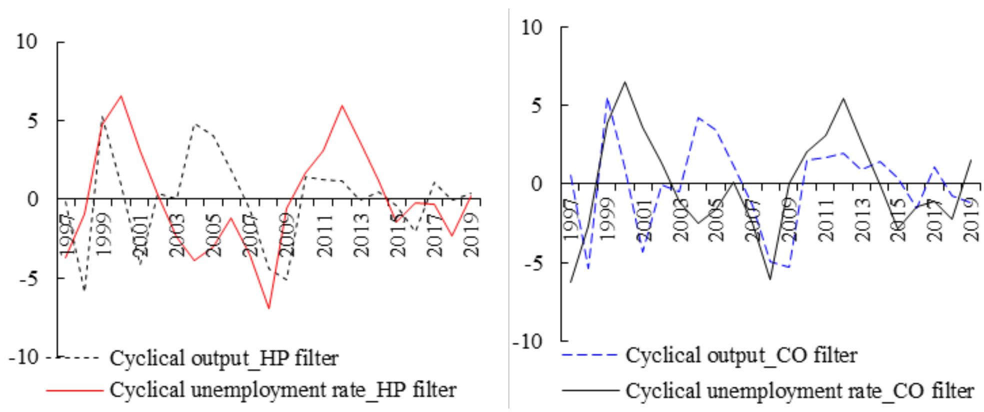
$$w_{(1/n)}(\lambda_s) = \frac{-1}{\sqrt{n}} \left(\frac{e^{i\lambda_s}}{1 - e^{i\lambda_s}} \right) \tag{5}$$

The leakage from the low frequency expressed in Eq. (5) can be removed by de-trending in the frequency domain, leaving an asymptotically unbiased $\dfrac{1}{1 - e^{i\lambda_s}} w_v(\lambda_s)$ estimate over the non-zero frequencies. This estimate is $\sqrt{n}$-consistent and has a good finite samples series.

More importantly, the smoothing parameter of the HP filter is set to 100 equivalent to the value of 6.25, as suggested by Ravn and Uhlig (2002), to generate robust filtered data from annual series, whereas the maximum (s) and minimum (e) values of the CO filter are set to 2 and 8 (frequencies 0.25 and 1 respectively) to retain the oscillation periods in our annual data, congruent to the business cycle.[7]

Figure 1 plots the evolution of the cyclical components of the unemployment and output (i.e. real GDP) series based on the HP and CO filters. In both cases, the three notable sharp troughs (peaks) in the cyclical output (cyclical unemployment) coincide with major global events such as (i) the 1997 Asian currency crisis, (ii) the unwinding of the speculative tech bubble (i.e. the dot.com bubble) in 2000/2001, and (iii) the synchronized fall in global economic growth during the 2007/08 recession.

Observably, cyclical unemployment exhibits a much higher magnitude compared to cyclical output, corroborating the persistently high unemployment rate in the Free State province, particularly during an economic expansion. Also, the inverse relationship between the cyclical output and cyclical unemployment is less clear-cut during the 25 years being studied, as the cyclical variables often cross at values different from zero, implying that a positive output gap is uncorrelated to a negative unemployment gap (or vice versa).



Source: author's estimation using EViews 11

Figure 1. *Estimated cyclical components of real output and unemployment using the HP and CO filters (baseline models)*

Moreover, to a large extent, there is evidence of a dominant positive correlation between unemployment and output growth in the Free State province, especially during the periods of 1998–2000 and 2009–2013, suggesting that the inverse unemployment–output nexus regularity theorized by Okun's law does not hold,

7 Refer to the recommended procedure for applying the Corbae–Ouliaris frequency-domain filter to time series, available at: http://fmwww.bc.edu/repec/bocode/c/couliari.html.

implying that an increase in economic activity level during these periods of economic expansion does not create the much-needed job opportunities in the provincial labour market. This inference concretely supports the evidence of the jobless growth phenomenon found in South Africa in the past decade, as stated earlier (see e.g. Phiri, 2014; Marinkov and Geldenhuys, 2007).

4. The Model: Non-Linear Autoregressive Distributed Lag (NARDL) Framework

To examine and validate the existence of an asymmetric relationship between cyclical output and cyclical unemployment, we rely on the gap specification of Okun's law, which links the activities in the goods market to the labour market, as well as captures movements in unemployment around the natural rate of unemployment owing to fluctuations in actual output around its trend growth over the business cycle, as stated earlier. Moreover, the NARDL modelling framework introduced by Shin et al. (2014) is most suitable for our empirical analysis because it effectively uncovers the "hidden cointegration" existing between positive and negative components of the cyclical variables (Granger and Yoon, 2002) and efficiently deals with the misspecification bias due to endogeneity or serial correlation by allowing the inclusion of an optimal number of lagged variables into the system.

For our application, consider an inverse relationship between the cyclical components of unemployment and output simply defined as:

$$\begin{aligned} u_t - u_t^* &= \alpha_0 + \beta(y_t - y_t^*) + \varepsilon_t \qquad \beta < 0 \\ \equiv u_t^g &= \alpha_0 + \beta y_t^g + \varepsilon_t \end{aligned}, \tag{6}$$

where: y_t and u_t are observed real output (i.e. actual output) and unemployment rate respectively; y_t^* is the potential output, and u_t^* is the natural rate of unemployment; y_t^g captures the cyclical output $(y_t - y_t^*)$; u_t^g accounts for cyclical unemployment $(u_t - u_t^*)$;[8] β denotes the Okun's coefficient; ε_t is the stochastic error term.

Next, a dynamic NARDL model is constructed using the specified model in Eq. (1) based on the unrestricted ARDL (p, q) model in the form:

$$\Delta u_t^g = \alpha_0 + \beta_1 u_{t-1}^g + \beta_2 y_{t-1}^g + \lambda_1 \sum_{k=1}^{p} \Delta u_{t-k}^g + \lambda_2 \sum_{k=1}^{q} \Delta y_{t-k}^g + \delta ECT_{t-1} + \xi_t, \tag{7}$$

8 In this paper, the macro-variables cyclical output, cyclical unemployment, and cyclical components are interchangeably referred to as output gap, unemployment gap, and gap variables respectively.

where Δ is the first-difference operator, λ_1 and λ_2 are the short-run coefficients with β_2 being the long-run coefficient (normalized on β_1), ξ_t is the normally distributed residual term such that $\xi_t \sim IID(0,\sigma^2)$, and δ is the coefficient of the one-period lagged error correction term ECT_{t-1}, which measures the adjustment speed of the system to its long-run equilibrium following a shock. Notably, the coefficient of the ECT_{t-1} term is expected to be negative and statistically significant to confirm the existence of a long-run relationship among variables.

However, the symmetric ARDL model specified in Eq. (2) is unsuitable to achieve our empirical objective, as it assumes that changes in cyclical unemployment are linearly related to fluctuations in cyclical output. Following Shin et al. (2014), short-run and long-run asymmetries are introduced into Eq. (2) by defining the potential output y_t^g as a k×1 vector consisting of multiple regressors such that $y_t^g = y_t^g + y_t^{g+} + y_t^{g-}$, yielding an asymmetric long-run regression expressed as:

$$u_t^g = \beta^+ y_t^{g+} + \beta^- y_t^{g-} + \xi_t \qquad \xi_t \sim N(0,\sigma^2)\,, \tag{8}$$

where β^+ and β^- are nonlinear error correction terms associated with asymmetric long-run parameters, y_t^{g+} and y_t^{g-} are the partial sum processes of positive and negative changes in cyclical output, and y_t^g is defined as:

$$\begin{aligned} y_t^{g+} &= \sum_{k=1}^{t} \Delta y_k^{g+} = \sum_{k=1}^{t} \max(\Delta y_k^g, 0) \\ y_t^{g-} &= \sum_{k=1}^{t} \Delta y_k^{g-} = \sum_{k=1}^{t} \max(\Delta y_k^g, 0) \end{aligned} \tag{9}$$

The decomposition of cyclical output into associated positive and negative changes in Eq. (9) captures the effects of differentiated output shocks on the unemployment rate during economic upswings (expansion) and downswings (recession) over the business cycle.

Substituting Eq. (9) into Eq. (7) yields a dynamic nonlinear ARDL (p, q) model consisting of an error correction mechanism represented as:

$$\begin{aligned} \Delta u_t^g = {} & \alpha_0 + \sum_{k=1}^{p} \rho_i u_{t-k}^g + \theta_k^+ y_{t-k}^{g+} + \theta_k^- y_{t-k}^{g-} + \sum_{k=1}^{p-1} \varphi_i \Delta u_{t-k}^g + \sum_{k=0}^{q-1} (\pi_k^+ y_{t-k}^{g+} + \pi_k^- y_{t-k}^{g-}), \\ & + \mu ECT_{t-1} + \omega_t \qquad \text{for} \qquad k = 1,\ldots,q \end{aligned} \tag{10}$$

where θ_k^+ and θ_k^- are the asymmetrically distributed lagged parameters, φ is the autoregressive parameter, and ω_t is an *IID* process with zero mean and constant variance, σ^2. Parameters ρ, θ_k^+, and θ_k^- are long-run coefficients, while π_k^+ and π_k^-

are short-run coefficients. In this case, $\beta^{g+} = -\theta^{+} / \rho$ and $\beta^{g-} = -\theta^{-} / \rho$ are the asymmetric long-run coefficients. The reliable estimates of $\theta_k^{+}, \theta_k^{-}, \pi_k^{+}$, and π_k^{-} are estimated using the standard ordinary least square method.

To account for the impacts of structural changes that can influence the asymmetric cyclical unemployment–cyclical output relationship, we add dummy variables into the specified dynamic NARDL model in Eq. (10), which is re-estimated as:

$$\begin{aligned}\Delta u_t^g = &\sum_{k=1}^{p} \rho_i u_{t-k}^g + \theta_k^{+} y_{t-k}^{g+} + \theta_k^{-} y_{t-k}^{g-} + \gamma_1 D_{t-k} + \sum_{k=1}^{p-1} \varphi_i \Delta u_{t-k}^g + \sum_{k=0}^{q-1} (\pi_k^{+} y_{t-k}^{g+} + \pi_k^{-} y_{t-k}^{g-}) \\ &+ \gamma_2 \Delta D_{t-k} + \mu ECT_{t-1} + \omega_t \qquad \omega_t \sim N(0, \sigma^2)\end{aligned}, \quad (11)$$

where γ_1 and γ_2 are the coefficients of the dummy variables identified based on the breakpoint unit root test and constructed for the periods of 2001–2009 and 2007–2012, while D_t denotes break dates such that $D_t = 1$ for $t \geq BD$ and 0 otherwise.

Table 1. *Results of the performed unit root tests on the cyclical variables y_t^g yc_t^g, and u_t^g*

Unit root test	HP filter		CO filter	
	Intercept	Intercept & trend	Intercept	Intercept & trend
DF-GLS				
y_t^g	-4.21*	-4.24*	-4.12*	-4.10*
u_t^g	-2.16**	-2.25	-2.36**	-2.65
Δu_t^g		-3.43**	-3.07*	-3.30**
yc_t^g	4.04*	4.05*	-3.94*	-3.89*
PP				
y_t^g	-4.07*	-3.95**	-4.05*	-2.99***
u_t^g	-2.48	-2.44	-3.94**	-2.87
Δu_t^g	-3.28**	-6.98*	--	-7.19*
yc_t^g	-3.89*	-3.78**	-3.88*	-3.77**
Breakpoint				
Innovative outlier (DF min-*t*, *F*-stat.)				
y_t^g	-4.34*** [2004]	-6.50* [2007]	-4.22*** [2004]	4.63 [2007]
Δy_t^g	-8.39* [2001]		8.25* [2001]	7.61* [2002]
u_t^g	-5.82* [2008]	-4.99*** [2008]	-5.39* [2008]	-5.32** [2008]
Δu_t^g		-5.80* [2008]		
yc_t^g	-4.24*** [2004]	-6.46* [2007]	-4.12 [2004]	-4.04 [2001]
Δyc_t^g	-8.151* [2001]		-7.99* [2001]	-7.21* [2002]

Unit root test	HP filter		CO filter	
	Intercept	Intercept & trend	Intercept	Intercept & trend
Additive outlier (DF min-t, F-stat.)				
y_t^g	-4.62** [2003]	5.09*** [2006]	-4.86* [2009]	-4.85 [2010]
Δy_t^g	[2003]	-7.68* [2001]		-7.67* [2010]
u_t^g	-5.26* [2006]	-4.50 [2005]	-5.62* [2005]	-5.50** [2005]
Δu_t^g		-7.72* [2008]		
yc_t^g	-4.61** [2003]	-5.03* [2006]	-4.66** [2009]	-4.63 [2009]
Δyc_t^g		-7.38* [2001]	-7.38* [2001]	-7.34* [2002]

Notes: *, **, and *** denote the significance of associated p-values at 1%, 5%, and 10% levels respectively. Breaks are chosen based on Dickey–Fuller min-t with optimal lag length (up to 4) being selected using F-statistics. Parameters y_t^g and u_t^g denote cyclical output and cyclical unemployment.

Finally, the bound-testing procedure proposed by Pesaran et al. (2001) to establish whether variables are cointegrated (i.e. long-run relationship) is performed using the dynamic NARDL model specified in Eq. (11). Here, the bound-testing procedure is based on a modified F-test of the joint null, $\hat{\rho} = \theta_k^+ = \theta_k^- = 0$. The standard Wald test is used to determine the presence of an asymmetric relationship between the two cyclical variables in both the long run ($H_0 : \hat{\theta} = \theta_k^+ = \theta_k^-$) and the short run (either as $H_0 : \pi_k^+ = \pi_k^-$ for all $i = 0,\dots q-1$ or as $H_0 : \Sigma_{i=0}^{q-1}\pi_k^+ = \Sigma_{i=0}^{q-1}\pi_k^-$).[9]

5. Empirical Results and Discussion

5.1. Unit Root Test

Typically, the specified NARDL model in Eq. (11) would be considered invalid if any of the interested variables are $I(2)$ stationary. Thus, the stationarity properties (i.e. order of integration) of the chosen cyclical variables were established using the augmented Dickey–Fuller GLS (Elliot et al. 1996), the Phillips–Perron (1989; hereafter PP), and the breakpoint (with DF min-t for capturing breaks in data owing to innovation and additive outliers) unit root tests.[10]

The results of the implemented unit root tests reported in *Table 1* generally show that the generated cyclical components of total unemployment, real output, and real GDP per capita are mostly stationary in level, $I(0)$, or require first differencing,

9 Carrying out the Wald test on the joint null of $\pi_k^+ = \pi_k^-$ and additive symmetry, $\Sigma_{i=0}^{q-1}\pi_k^+ = \Sigma_{i=0}^{q-1}\pi_k^-$, it produces a strong form and a weak form of short-run symmetry respectively.

10 The BP unit root test allows for a structural break in the deterministic trend to be endogenously determined at unknown dates. This test was developed based on the theoretical contributions of Perron (1989), Vogelsang and Perron (1998), Zivot and Andrews (1992), and Banerjee et al. (1992).

I(1). In addition, the breakpoint (BP) unit root test result indicates structural breaks in these cyclical variables that coincide with notable global events or shocks, justifying the inclusion of dummy variables in the computed nonlinear models.

5.2. Cointegration Analysis

Having found that the cyclical components of unemployment and output are either I(0) or I(1) stationary, the next step is to apply the bound-testing procedure to determine whether these gap variables are asymmetrically (or symmetrically) cointegrated, that is, whether they exhibit nonlinear or linear relationships in the long run, as stressed in the previously discussed empirical literature.

Table 2. *Results of bounds test for cointegration in the baseline NARDL models with HP- and CO-filtered cyclical variables*

Dependent variable: ΔU_t^g		Function: $\hat{u}_t^g = f(\hat{y}_t^{g+}, \hat{y}_t^{g-})$			
Detrending method	Dynamic specification	F-statistic F_{PSS}		Conclusion	
HP filter	NARDL (2,2,1,1)	20.712		Asymmetric cointegration exists	
CO filter	NARDL (2,2,1,1)	7.6087		Asymmetric cointegration exists	
Asymptotic critical values (CVs) for k = 3.					
1%		5%		10%	
I (0)	I (1)	I (0)	I (1)	I (0)	I (1)
5.333	7.063	3.71	5.018	3.008	4.15

Notes: I (0) and I (1) denote the upper bound and lower bound levels respectively. Asymptotic critical values (CVs) for nonlinear models are evaluated based on Case III (unrestricted constant and restricted), as in Pesaran et al. (2001).

To test for cointegration, two dynamic NARDL models as specified in Eq. (11) were computed, with one consisting of cyclical unemployment and cyclical output (based on real GDP) variables produced by the HP filter and the second one comprising cyclical components of unemployment and real output generated using CO filter.[11] Also, each estimated model consists of optimal lags of 2 selected based on the Akaike information criterion (AIC).

In what follows, the bound testing procedure is applied to test the joint F-test for the null of $\hat{\rho} = \hat{\theta}_k^+ = \hat{\theta}_k^+ = 0$ (see Pesaran et al., 2001), and the cointegration analysis results reported in *Table 2* show that the values of the joint F-statistics (F_{PSS}) exceed the upper bound critical values at a 1% significance level, indicating the existence of an asymmetric relationship between the cyclical unemployment and

11 The computed NARDL models here are taken as the baseline models.

cyclical output (generated using the HP and CO filtering methods) in the estimated dynamic nonlinear models.[12]

5.3. Evidence of Asymmetric Cyclical Unemployment–Cyclical Output Nexus

Given the evidence of an asymmetric long-run relationship between cyclical unemployment and cyclical output in the estimated baseline nonlinear models, we assess the fitness of the asymmetric dynamic model by testing for the null hypothesis of long-run symmetry ($W_{LR} : \theta_k^+ = \theta_k^-$) and short-run symmetry ($W_{SR} : \Sigma_{i=0}^{q-1}\hat{\pi}_k^+ = \Sigma_{i=0}^{q-1}\hat{\pi}_k^-$) using the Wald statistic following an asymptotic chi-square (χ^2) distribution.

The result of the performed Wald tests for the baseline nonlinear models is reported in *Table 3* (lower panel), and it firmly rejects the null hypothesis of a symmetric relationship between cyclical unemployment and output in the long run and the short run. In particular, the *F*-statistics for long-run (W_{LR}) and short-run (W_{SR}) asymmetries are 32.5 and 5.29, respectively, in the baseline nonlinear model with HP-filtered cyclical variables, while the respective values of W_{LR} and W_{SR} are 11.6 and 8.80 in the model with CO-filtered cyclical variables. Overall, the results of the Wald statistics for W_{LR} and W_{SR} in the computed baseline models are statistically significant at a 1% significance level, validating the existence of the asymmetric cyclical unemployment–cyclical output nexus in the Free State province.

As regards the asymmetric inverse relationship between cyclical unemployment and cyclical output in the long run, the results in *Table 3* (upper panel) show negative and statistically significant coefficients for the one-period lagged cyclical unemployment and the "positive" changes in cyclical output in the baseline models, indicating the presence of a "partial" long-run asymmetric cyclical unemployment–cyclical output nexus in the Free State province. Interpretively, the coefficients of the long-run cyclical output show that a one percent increase in cyclical output will reduce cyclical unemployment to the range between 0.72 (significant at a 1% critical level) and 0.87 percentage points (significant at a 10% critical level), conditioned on periods of sustained economic expansion. Our result aligns with the findings of Marinkov and Geldenhuys (2007), showing that a one percent increase in cyclical output is associated with a reduction in cyclical unemployment by 0.77 percentage points for South Africa, yet being at odds with Kavase and Phiri (2020), who found no evidence of an inverse asymmetric cyclical unemployment–cyclical output nexus in both the long and short run in the Free State province.

12 The existence of the asymmetric long-run relationship between cyclical unemployment and cyclical output observed in the baseline models is further reinforced by the negative and statistically significant coefficients of the one-period lagged error correction terms (ECT_{t-1}) presented in *Table 2*.

Further, our finding on the existence of partial asymmetric and inverse cyclical unemployment–cyclical output nexus is reinforced by the highly significant and negative long-run coefficient estimates L_y^+ of -0.57 and -0.53 for positive cyclical outputs in the baseline model with HP- and CO-filtered gap variables respectively. This inference implies that an economic upturn between 1.73 and 1.88 percent is necessary to reduce unemployment by one percent. Also, this empirical result affirms the existence of Okun's relationship with a trade-off ratio of 2:1 in the Free State province, in keeping with documented findings in previous studies (see e.g. Adanu, 2005; Freeman, 2000; Gordon, 1998).

Interestingly, the estimated long-run coefficients L_y^- for the negative cyclical outputs in the nonlinear models with HP- and CO-filtered variables are 0.41 and 0.31 respectively, reflecting a contradictory positive relationship between negative cyclical output and cyclical unemployment, indicating that a decline in negative output gap between 3.8 and 4.3 percent will lower cyclical unemployment by one percent in the long run; however, these effects are statistically insignificant. This result corroborates those of Kavase and Phiri (2020), who found an insignificant positive relationship between the negative output gap and the unemployment gap in the Free State province, with a percentage decrease in the output gap reducing the unemployment gap by 1.66 percent in the long run.[13]

Although the above finding may be counterintuitive, it still explains the observed positive co-movement between economic growth and unemployment in *Figure 1*, which reflects certain periods (e.g. 1997–1999 and 2009–2013) in the provincial business cycle when there is a simultaneous rise in both economic activity level and unemployment rate.

Overall, in all cases, the estimated short-run coefficients for cyclical variables, Δy_t^{g+} and Δy_{t-1}^{g+}, are negative and statistically significant in both nonlinear (baseline) models, suggesting that changes in cyclical output exert strong contemporaneous effects on cyclical unemployment in the short run in the province being studied. More specifically, a one percent rise in cyclical output (i.e. positive output gap) contemporaneously reduces cyclical unemployment between 1.55 and 1.63 percentage points during an economic downswing. Further, the negative and statistically significant coefficients for one-period lagged short-run cyclical output variables (i.e. Δy_t^{g+} and Δy_{t-1}^{g+}) in the baseline models affirm the existence of asymmetric short-run effects, suggesting that a one percentage increase (decrease) in cyclical output can reduce (raise) cyclical unemployment by 0.52 (+0.83) during an economic downturn (upturn) in the short run, after the impact of the output shock (say about a year).

13 Similar findings on insignificant positive cyclical output–cyclical unemployment nexus in the long run have been documented in previous works exploring the presence of asymmetric Okun's relationship in South Africa: see, for example, Sere (2020) and Marinkov and Geldenhuys (2007).

Taken together, so far, our results suggest the existence of a short-run asymmetric cyclical unemployment–cyclical output relationship in the Free State province during economic downturns and upswings, over the business cycle. Similar findings about short-run asymmetric Okun's relationship have been documented in some South African provinces (Kavase and Phiri, 2020) and in South Africa (Mazorodze and Siddiq, 2018; Marinkov and Geldenhuys, 2007).

As anticipated, the statistically significant and negative coefficients for the ECT_{t-1} terms in the nonlinear baseline models presented in *Table 2* indicate the reversion of the bi-variate nonlinear systems to their long-run equilibrium (or steady state) in the presence of an external shock or an economic perturbation. Additionally, this result reaffirms the previously discussed finding of the existence of asymmetric cointegration between cyclical unemployment and cyclical output in the long run in the focal South African province.

Conversely, the absolute value of the speed of the adjustment μ_S ranges between 1.35 and 1.52 in the baseline models,[14] which is greater than unity but falls within the acceptable range of -1 and -2 to ensure the stability and convergence of the systems consisting of cointegrated cyclical variables, in the aftermath of an economic perturbation or external shock (Johansen, 1995). The implication is that, in the presence of an external shock to the bivariate system, the two cyclical variables will converge to their steady-state equilibrium (or long-run relationship) rapidly (say, in less than a year) following an oscillatory process, instead of the usual monotonic gradual reversion process to long-run equilibrium (Narayan and Symth, 2006).

In all cases, the statistically significant and negative coefficients of the dummy variables in the computed asymmetric models accentuate the vulnerability of the provincial economy and labour market activities to exogenous and structural endogenous shocks, which exert significant influence on changes in cyclical output, as well as the responsiveness of unemployment rate to output shocks. This result heavily stressed the notion that for effective policymaking the government needs to be cognizant of the significant impact of structural shocks on the stability of the existing asymmetric cyclical unemployment–cyclical output trade-off, especially at the provincial level.

Last but not least, the result of the performed diagnostic tests (*Table 3*, lower panel) explicitly shows that the statistic values for the JB normality test, the BG serial correlation test, the ARCH tests, and the RESET tests are statistically insignificant, affirming that the residuals of re-specified dynamic asymmetric models are free from irregular error distribution, serial correlation, heteroscedasticity, and specification bias due to the incorrect functional form of Okun's relationship.

14 Kavase and Phiri (2020: 70) reported relatively large and significant negative coefficients for the speed of adjustments for some South African provinces. Also, see Narayan and Symth (2006: 339) for related results and justification.

The results of the CUSUM and CUSUMSQ tests plotted in *figures 2–3* confirm the stability of estimated coefficients since the regression lines are confined within the 5% critical bounds of parameter stability in the baseline dynamic nonlinear models. Finally, the results of autocorrelation (AC) and partial autocorrelation (PAC) tests performed on both the correlogram of residuals and residuals squared derived from the baseline NARDL models show that these residuals are well-behaved, stationary, and not serially correlated (i.e. they have no pattern) with statistically insignificant *p*-values (up to 12 lags).[15]

Table 3. *Estimation results of the baseline NARDL models for the cyclical unemployment–cyclical output relationship*

Dependent variable: ΔU_t^g							
NARDL model with HP-filtered gap variables				NARDL model with CO-filtered gap variables			
Var.	Coeff.	S. E	*t*-ratio (*p*-values)	Var.	Coeff.	S. E	*t*-ratio (*p*-values)
Constant	39.62	6.71	5.90 (0.00)	Constant	32.07	9.12	3.51 (0.00)
u_{t-1}^g	-1.52	0.18	-8.13 (0.00)	u_{t-1}^g	-1.35	0.25	-5.35 (0.00)
y_{t-1}^{g+}	-0.87	0.24	-3.56 (0.00)	y_{t-1}^{g+}	-0.72	0.36	-1.97 (0.08)
y_{t-1}^{g-}	0.40	0.24	1.67 (0.12)	y_{t-1}^{g-}	0.31	0.33	0.93 (0.37)
L_y^+	-0.57	0.15	-3.68 (0.00)	L_y^+	-0.53	0.26	-1.97 (0.07)
L_y^-	0.26	0.14	1.86 (0.09)	L_y^-	0.23	0.22	1.01 (0.33)
Δu_{t-1}^g	-0.92	0.14	6.44 (0.00)	Δu_{t-1}^g	-0.93	0.20	4.48 (0.00)
Δy_t^{g+}	-1.63	0.29	-5.54 (0.00)	Δy_t^{g+}	-1.55	0.45	-3.39 (0.00)
Δy_t^{g-}	1.86	0.31	5.89 (0.00)	Δy_t^{g-}	1.85	0.44	4.13 (0.00)
Δy_{t-1}^{g+}	-0.52	0.17	-3.07 (0.01)	Δy_{t-1}^{g+}	-0.41	0.25	-1.62 (0.13)
Δy_{t-1}^{g-}	-0.83	0.35	-2.37 (0.04)	Δy_{t-1}^{g-}	-0.56	0.46	-1.20 (0.26)
$\gamma_1 D_{t-1}$	-15.34	2.75	-5.56 (0.00)	$\gamma_1 D_{t-1}$	10.39	3.17	-3.27 (0.00)
$\gamma_2 \Delta D_t$	-9.69	2.87	-3.37 (0.04)	$\gamma_2 \Delta D_t$	-7.57	1.93	-3.90 (0.00)
ECT_{t-1}	-1.52	0.14	10.51 (0.00)	ECT_{t-1}	-1.35	0.19	-7.02 (0.00)
Model diagnostics (lower panel)							
W_{LR}	32.56 (0.00)	W_{SR}	5.29 (0.02)	W_{LR}	11.60 (0.00)	W_{SR}	8.80 (0.00)
R^2	0.95	$\bar{R}^2$	0.89	R^2	0.87	$\bar{R}^2$	0.74
F-stat.	17.47 (0.00)	DW	2.04	*F*-stat.	6.40 (0.00)	DW	2.07
χ^2_{SC}	1.30 (0.52)	χ^2_{NOR}	3.67 (0.15)	χ^2_{SC}	0.80 (0.66)	χ^2_{NOR}	2.96 (0.22)
$\chi^2_{HET-ARCH}$	0.76 (0.38)	χ^2_{EF}	2.62 (0.14)	$\chi^2_{HET-ARCH}$	1.30 (0.25)	χ^2_{EF}	2.12 (0.18)

Notes: Subscripts "+" and "–" denote the positive and negative partial sum processes respectively. L_y^+ (i.e. $\hat{\beta}^{g+} = -\hat{\theta}_k^+ / \hat{\rho}$) and L_y^- (i.e. $\hat{\beta}^{g+} = -\hat{\theta}_k^- / \hat{\rho}$) are the long-run coefficients associated with positive and negative changes in cyclical output y^{g+} respectively. W_{LR} and W_{SR} refer to the Wald test for long-run symmetry (i.e. $L_y^+ = L_y^+$) and the additive short-run symmetry condition (i.e.,$\Sigma_{i=0}^{q-1}\hat{\pi}_k^+ = \Sigma_{i=0}^{q-1}\hat{\pi}_k^-$).

15 To save space, the AC and PAC test results are not reported here but are available upon request from the author.

χ^2_{SC}, χ^2_{H-ARCH}, χ^2_{NOR}, χ^2_{N}, and χ^2_{FF} are the LM tests for serial correlation, heteroscedasticity, normality, and function form (Ramsey's RESET) respectively. Corresponding p-values are in parentheses.

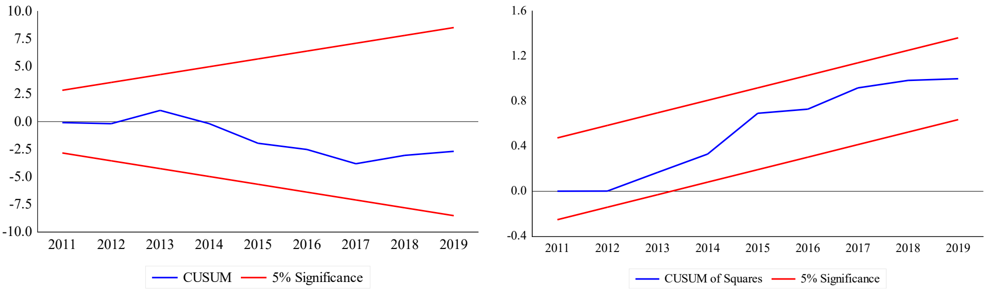


Figure 2. *CUSUM and CUSUMSQ of the baseline NARDL model with HP-filtered cyclical series*

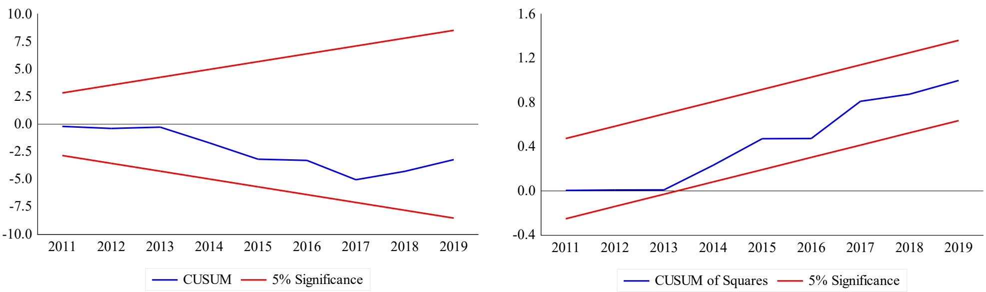


Figure 3. *CUSUM and CUSUMSQ of the baseline NARDL model with CO-filtered cyclical series*

Robustness: How Reliable Is the Asymmetric Okun's Relationship Using Provincial Data?

Hitherto, empirical results in the preceding analyses indicate the existence of the asymmetric cyclical unemployment–cyclical output nexus in the Free State province over the business cycle under consideration. However, intuitively, the results of the computed nonlinear baseline models are expected to be sensitive to specification bias, for instance, when a different measure of provincial output is used. Thus, for robustness, the dynamic asymmetric model specified in Eq. (7) is re-estimated with cyclical components of real GDP per capita (yc_t) and total unemployment (u_t) generated using the HP and CO filters.[16]

16 *Figure A3* in the *Appendix* plots the graphs of the growth rates of cyclical real GDP per capita and cyclical unemployment estimated using the HP and CO filters. Noticeably, these graphs exhibit an oscillatory pattern similar to *Figure 1*.

Table 4. *Results of bounds test for nonlinear cointegration in the sensitivity analysis NARDL models with HP- and CO-filtered cyclical variables*

Dependent variable: ΔU_t^g		Function: $\hat{u}_t^g = f(\hat{y}c_t^{g+}, \hat{y}c_t^{g-})$			
Detrending method	Dynamic specification	F-statistic (F_{PSS})		Conclusion	
HP filter	NARDL (2,2,1,1)	14.17898		Cointegration – asymmetric relationship exists	
CO filter	NARDL (2,2,2,0)	6.516974		Cointegration – asymmetric relationship exists	
Asymptotic CVs (k = 3) for the estimated models with HP and CO-filtered series					
1%		5%		10%	
I (0)	I (1)	I (0)	I (1)	I (0)	I (1)
5.333	7.063	3.71	5.018	3.008	4.15

Note: See Table 2.

Same as before, the result of the performed unit root tests shows that the cyclical variables yc_t and u_t are I(1) stationary (see *Table 1*). Next, we determine whether u_t and yc_t are cointegrated, making use of the bounds testing procedure; results of the cointegration analysis presented in *Table 4* indicate the existence of an asymmetric long-run relationship between the two cyclical variables since the F-statistics of 14.17 and 6.51, obtained from the re-estimated asymmetric models with HP-filtered and CO-filtered gap variables respectively, exceed the upper bound critical values (at a 1% significance level).

Based on the evidence of asymmetric cointegration between cyclical variables in the re-estimated nonlinear models, we applied the standard Wald test to ascertain the presence of asymmetries among the positive and negative components of the cyclical unemployment and cyclical output in the long run (W_{LR}) and short run (W_{SR}). Specifically, the Wald test results displayed in *Table 5* (lower panel) reveal statistically significant Wald statistics (at a 1% significance level), which rejects the null hypothesis of long-run and short-run symmetries but affirms the existence of asymmetric cyclical unemployment–cyclical output relationships in both the long and the short run. This finding inference corroborates previous results from the nonlinear baseline models (see *Table 3*). Thus, we can firmly conclude the existence of the asymmetric cyclical unemployment–cyclical output nexus for the Free State province both in the long run and the short run.

On the other hand, the empirical results presented in *Table 5* show that changes in cyclical outputs (i.e. positive and negative) have strong asymmetric long- and short-run effects on cyclical unemployment in the re-estimated models. Based on statistically significant results, there is concrete evidence of a "partial" asymmetric long-run relationship between the cyclical output and cyclical unemployment, specifically in the nonlinear model with HP-filtered gap variables. In particular, the empirical result suggests that a percentage rise in "positive" cyclical output will reduce cyclical unemployment by 0.70 percentage points. Meanwhile, the estimated long-run coefficient (L_y^+) of the "positive" cyclical output is -0.49, implying that an economic upturn of about 2.03 percent is necessary to reduce unemployment by one percent in the focal South African province. This finding agrees with previous evidence for the existence of an Okun's relationship with a trade-off ratio of 2:1, underscoring the cyclical output–cyclical unemployment nexus stressed in some studies (see Adanu, 2005; Attfield and Silverstone, 1998; Moosa, 1997, among others).

In connection with short-run asymmetries, we can observe strong contemporaneous effects of cyclical output on cyclical unemployment, in keeping with the result of the computed nonlinear baseline models. More specifically, the result of the re-estimated models reveals that a one percent increase in "positive" cyclical output would contemporaneously lower cyclical unemployment between 1.06 and 1.49 percentage points in the short run, and these effects are statistically significant at a 1% critical level. Equally, in the short run, a percentage increase in positive (negative) cyclical output would lower (raise) cyclical unemployment by 0.46 (0.99), and these asymmetric impacts are highly significant at a 1% critical level.

As expected, in all cases, the coefficients of the ECT_{t-1} terms are negative and highly significant, indicating the reversion of re-estimated nonlinear models to long-run equilibrium, after an external shock. Same as the baseline models, the adjustment speed is -1.42 (HP-filtered gap model) and -1.14 (CO-filtered gap model), suggesting a rapid and oscillatory reversion process to a long-run equilibrium. On the other hand, parameters γ_1 and γ_2 denoting the dummy variables are negative and statistically significant (mostly at a 1% critical level) in the re-estimated nonlinear model, accentuating the susceptibility of the evolution of asymmetric cyclical output–cyclical unemployment nexus found in the Free State province to structural endogenous and/or external shocks.

Lastly, considering the result of the performed diagnostic tests (*Table 5*, lower panel), the statistic values for the JB normality test, BG serial correlation test, ARCH tests, and RESET tests are statistically insignificant, affirming that the residuals of re-specified dynamic asymmetric models are free from irregular error distribution, serial correlation, heteroscedasticity, and specification bias due to the incorrect functional form of Okun's relationship. The results of the CUSUM and CUSUMSQ tests plotted in *figures 4–5* indicate the absence of any instability of the parameter coefficients, as the regression lines are confined within the 5% critical bounds of parameter stability in the re-estimated dynamic nonlinear models. The results of autocorrelation (AC) and partial autocorrelation (PAC) tests performed on both the correlogram of residuals and residuals squared derived from the sensitivity analysis NARDL models show that these residuals are stationary and serially uncorrelated with statistically insignificant *p*-values (up to 12 lags).

Table 5. *Estimation results of the NARDL models for cyclical unemployment–cyclical output relationship (sensitivity analysis)*

Dependent variable: $\varDelta U^g_t$							
NARDL model with HP-filtered gap variables				NARDL model with CO-filtered gap variables			
Var.	Coeff.	S. E	t-ratio (p-values)	Var.	Coeff.	S. E	t-ratio (p-values)
Constant	36.03	7.63	4.71 (0.00)	Constant	21.45	7.35	2.91 (0.01)
u^g_{t-1}	-1.42	0.21	-6.75 (0.00)	u^g_{t-1}	-1.14	0.24	-4.69 (0.00)
yc^{g+}_{t-1}	-0.70	0.27	-2.53 (0.03)	yc^{g+}_{t-1}	-0.31	0.35	-0.91 (0.38)
yc^{g-}_{t-1}	0.50	0.27	1.81 (0.10)	yc^{g-}_{t-1}	0.40	0.35	-0.91 (0.23)
L^+_{cy}	-0.49	0.18	-2.63 (0.02)	L^+_{cy}	-0.27	0.30	-0.90 (0.38)
L^-_{cy}	0.35	0.17	2.04 (0.07)	L^-_{cy}	0.35	0.25	1.41 (0.18)
$\varDelta u^g_{t-1}$	-0.87	0.16	5.38 (0.00)	$\varDelta u^g_{t-1}$	-0.72	0.16	4.44 (0.00)
$\varDelta yc^{g+}_t$	-1.49	0.34	-4.30 (0.00)	$\varDelta yc^{g+}_t$	-1.06	0.25	-4.20 (0.00)
$\varDelta yc^{g-}_t$	1.71	0.35	4.88 (0.00)	$\varDelta yc^{g-}_t$	1.42	0.32	4.41 (0.00)
$\varDelta yc^{g+}_{t-1}$	-0.46	0.14	-3.11 (0.01)	$\varDelta yc^{g+}_{t-1}$	---	---	---
$\varDelta yc^{g-}_{t-1}$	-0.99	0.22	-4.49 (0.00)	$\varDelta yc^{g-}_{t-1}$	-0.94	0.28	-3.31 (0.00)
$\gamma_1 D_{t-1}$	-1.498	0.34	-4.30 (0.00)	$\gamma_1 D_{t-1}$	-7.99	3.11	-2.56 (0.02)
$\gamma_2 \varDelta D_t$	-10.41	3.43	-3.03 (0.01)	$\gamma_2 \varDelta D_t$	---	---	---
ECT_{t-1}	-1.42	0.163	-8.69 (0.00)	ECT_{t-1}	-1.14	0.19	-5.75 (0.00)
Model diagnostics (lower panel)							
W_{LR}	20.91 (0.00)	W_{SR}	10.15 (0.00)	W_{LR}	7.75 (0.00)	W_{SR}	4.22 (0.03)
R^2	0.89	$\bar{R}^2$	0.78	R^2	0.81	$\bar{R}^2$	0.67
F-stat.	7.97 (0.00)	DW	1.71	F-stat.	5.91 (0.00)	DW	2.10
χ^2_{SC}	2.64 (0.26)	χ^2_{NOR}	0.90 (0.63)	χ^2_{SC}	2.73 (0.25)	χ^2_{NOR}	0.27 (0.88)
$\chi^2_{HET-ARCH}$	1.72 (0.18)	χ^2_{EF}	4.16 (0.07)	$\chi^2_{HET-ARCH}$	1.34 (0.24)	χ^2_{EF}	0.26 (0.61)

Notes: See *Table 2*. Here: L^+_{cy} and L^-_{cy} represent the long-run coefficients associated with positive and negative changes in cyclical output (yc^{g+}) proxy as real per capita GDP respectively.

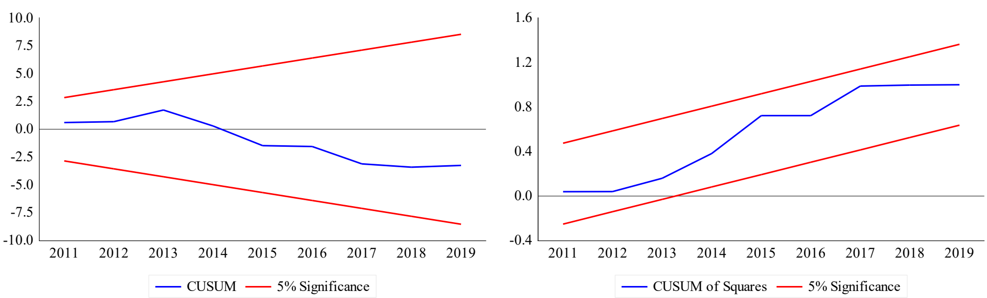


Figure 4. *CUSUM and CUSUMSQ of the sensitivity analysis NARDL model with HP-filtered cyclical variables*

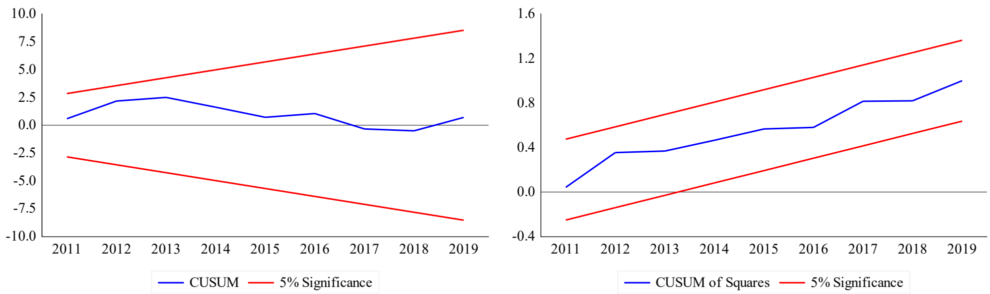


Figure 5. *CUSUM and CUSUMSQ of the sensitivity analysis NARDL model with CO-filtered cyclical variables*

6. Conclusions and Policy Recommendations

This paper contributes to two emergent strands of empirical literature testing for asymmetry in Okun's law and those investigating Okun's relationship at the regional (provincial or state) level. Specifically, we examine whether there is evidence supporting the asymmetric unemployment–output relationship in the Free State province (South Africa) employing the NARDL modelling approach to account for both long- and short-run asymmetries among variables in a coherent manner. In addition, the Hodrick–Prescott and Corbae–Ouliaris detrending techniques were applied to isolate the cyclical components of unemployment and output growth rates in the baseline non-linear models. For sensitivity analysis, the gap variable of real GDP per capita (a different measure of cyclical output) is substituted into the baseline models to determine the robustness of the obtained results. Irrespective of the detrending methods, we find the value of Okun's coefficient to be relatively stable around 2 and strong evidence for asymmetric Okun's relationship in the Free State province given the evidence of a statistically significant asymmetric cyclical unemployment–cyclical output relationship in both the long and short run when the effect of structural break is accounted for.

Summarily, our empirical results align with findings documented in the literature on the existence of nonlinear Okun's relationship, particularly in South Africa. Firstly, we find evidence of a "partial" long-run asymmetry given that only the coefficient of positive cyclical output exhibits a statistically significant inverse relationship with cyclical unemployment. Specifically, our results suggest that a 1 percent increase in positive cyclical output will lower cyclical unemployment between 0.70 and 0.87 percentage points, conditioned on a sustained economic upswing. Secondly, the statistically significant long-run coefficients reveal that a fall in unemployment by one percent in the Free State province requires an economic upswing between 1.88 to 2.03 percent, keeping in line with the proven empirical regularity of Okun's ratio of 2:1 documented in the growing literature. Thirdly, we find evidence of a strong contemporaneous effect of asymmetric changes in cyclical output on cyclical unemployment in the short run. To this end, the estimates of contemporaneous (i.e. lagged and one-period lagged) coefficients for both the positive and negative cyclical outputs are negative and statistically significant, implying that a one percent increase in positive cyclical output is associated with a decrease in cyclical unemployment between 1.63 and 1.06 percentage points, whereas a one percent decline in negative cyclical output would increase cyclical unemployment between 0.41 and 0.52 percentage points in the Free State province.

Our findings have crucial policy implications. In particular, the evidence of non-linear cyclical unemployment–output nexus explains the ineffectiveness of the nationally ratified pro-poor policies to spur economic growth and lower the prevailing high unemployment rate in South Africa and across its provinces. Meanwhile, the existence of long- and short-run asymmetries between unemployment and output accentuates the policy imperativeness for policymakers to take into account both supply and demand factors influencing economic activity and unemployment, which is typically overlooked during policymaking since the non-linear relationship between these macro-variables emphatically calls for adopting an integrated economic reform that would simultaneously tackle the problem of weak economic performance and rising unemployment rate, rather than the isolated piecemeal policy approach commonly used to deal with these macroeconomic problems individually.

Based on our findings, we put forward some policy prescriptions for both the provincial and national governments to implement macroeconomic reforms that proactively deal with the emergent nonlinear association between economic growth and unemployment, as well as improve the responsiveness of the latter to real output growth in South Africa. Firstly, to effectively address the problem of skill mismatch – the main driver of the unemployment rate in the country coupled with the prevailing highly skilled intensive domestic labour market that mostly absorbs educated workers –, it is imperative for the government (via the provincial economic development department) to proactively promote entrepreneurship and self-employment initiatives, particular among the youth population. Typically, these employment initiatives can be reinforced by providing access to public and private funding for the development of credible business plans and the establishment of small, medium, and micro-enterprises.

Secondly, given the rapid growth and increasing importance of the informal sector in the South African economy as a source of employment and an indirect contributor to economic growth, it is important for policymakers to remove market rigidities associated with, for example, obstructive bureaucratic red tapes to acquire trading (or business) permits, company registration fees, extended turn-around period for company/business registration, and stringent by-laws that hamper business operation in the informal sector. This policy strategy can lead to higher productivity and competition in the labour market, which in turn will increase the provincial contributions to the domestic economy by raising total productivity and lowering the unemployment rate at the national level.

Finally, to improve the responsiveness of the unemployment rate to economic growth in the short run, given the evidence of short-run asymmetry, the provincial government needs to intensify its investment spending on large infrastructure projects and preventative maintenance of public infrastructures (such as government buildings and road networks) since such effort creates transitory job opportunities, especially for the unemployed unskilled workers which form the largest portion of the total unemployment rate (DPWI, 2020). In the same vein, the government should deepen its efforts to efficiently implement its flagship public employment programmes (PEPs) to prevent moral hazard and rent-seeking problems (e.g. corruption and nepotism practices) associated with the selection of beneficiaries and stipulated participation period. This effort would improve the labour market absorption rate, in effect lowering unemployment and alleviating poverty, as the nationally-driven PEPs are mostly labour-intensive and prioritize skill development (Maphanga and Mazenda, 2019; McCord, 2004, 2005).

The scope of this study is limited by the paucity of reliable historical time series available for South African provinces, which is evidenced by the scanty provincial-specific studies in the voluminous literature assessing asymmetric Okun's law using provincial (or regional) data. As a result, the dynamic asymmetric model constructed for our analysis consists of few reliable macro-variables.

Future studies can build our work using a similar asymmetric model and incorporating some macroeconomic, fiscal, and institutional variables (for example, employees' working hours, changes in female labour participation, labour productivity, demographic trends, capacity utilization, and wage rate) found in existing country-specific studies as determinants of asymmetric Okun's relationship (see e.g. Ball et al., 2017; Knotek, 2012; Lee, 2000; Moosa, 1997; Prachowny, 1993). A typical empirical analysis would allow policymakers to better understand how unobserved macroeconomic and labour force conditions could influence asymmetric unemployment–output relationship across South African provinces, to develop and implement inclusive macro-policies that improve the responsiveness of unemployment to output growth.

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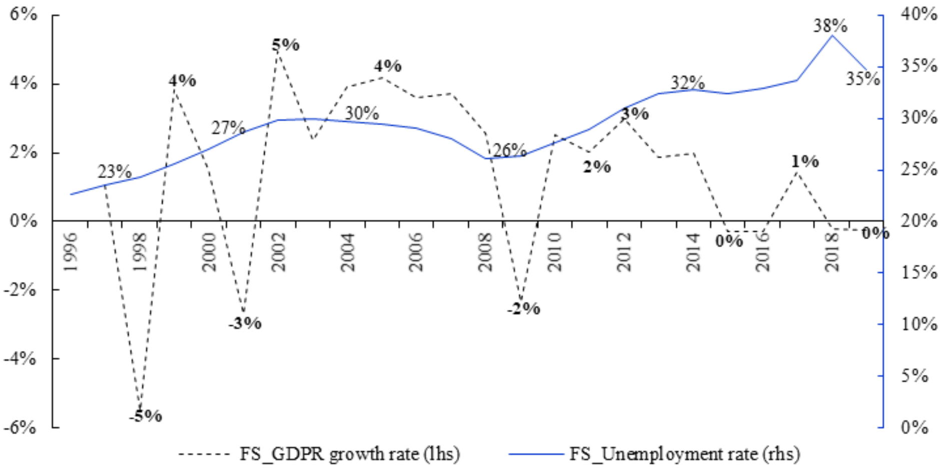
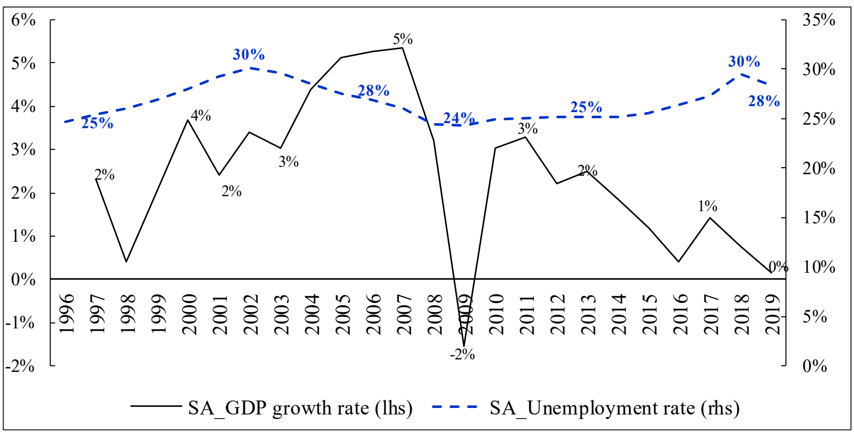
APPENDIX A.



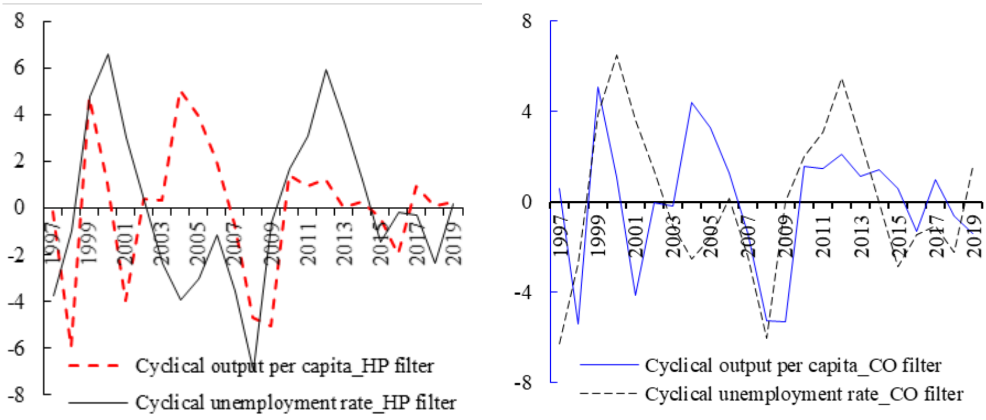
Source: StatsSA (2020); S&P Global Regional eXplorer (ReX); author's estimation

Figure A1. *Evolution of annual real GDP and unemployment growth rates in FS Province, 1996–2019*



Source: StatsSA (2020); S&P Global Regional eXplorer (ReX); author's estimation

Figure A2. *Evolution of annual real GDP and unemployment growth rates in South Africa, 1996–2019*



Source: author's estimation using EViews 11

Figure A3. *Estimated cyclical components of real output per capita and unemployment using the HP and CO filters (sensitivity analysis models)*

Table A1. *Summarized descriptive statistics for unobserved components of u_t and yc_t (1994–2019)*

	HP detrending method		CO filter detrending method	
Description	Cyclical unemployment (%)	Cyclical output (%)	Cyclical unemployment (%)	Cyclical output (%)
	$u_t^g = u_t - u_t^*$	$y_t^g = y_t - y_t^*$	$u_t^g = u_t - u_t^*$	$y_t^g = y_t - y_t^*$
Mean	-1.66e-14	2.24e-14	1.13e-15	7.63e-16
Median	-0.281	0.299	-0.062	0.606
Maximum	6.599	5.040	6.496	5.098
Minimum	-6.929	-5.963	-6.258	-5.402
Std. dev.	3.397	2.849	3.306	2.872
Skewness	0.198	-0.400	0.056	-0.496
Kurtosis	2.526	2.971	2.513	2.743
Jarque–Bera statistics (*p*-value)	0.366 (0.832)	0.616 (0.734)	0.238 (0.887)	1.006(0.604)
Sum	-3.81e-13	5.14e-13	2.35e-14	1.73e-14
Sum Sq. Dev.	253.885	178.606	240.575	181.542

Source: author's estimation using EViews 11

Note: *p*-values in parentheses.

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