



Adaptive Digital Notch Filter for Enhanced Stability in Grid-Connected Inverter Systems

Mohammed Moyed AHMED¹

¹ Department of ECE, JNTU, Hyderabad, INDIA,
e-mail: mmoyed@gmail.com

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Abstract: Grid-connected inverters with LCL filters are crucial components in renewable energy systems, but they face stability challenges due to varying grid impedances and resonant frequencies. This paper presents a novel adaptive digital notch filter designed to enhance the stability of such systems across a wide range of operating conditions. The proposed filter employs a three-step adaptive mechanism: resonance detection, determination of resonant frequency change direction, and dynamic notch frequency adjustment. A comprehensive stability analysis in the z -plane reveals the behavior of the resonant pole under different scenarios, informing the filter's design. The adaptive filter's performance was evaluated through simulations. Results demonstrate significant improvements in system stability compared to conventional fixed-frequency notch filters. In scenarios where the resonant frequency was lower than the initial notch frequency, the adaptive filter prevented current divergence and reduced the line current harmonic magnitude at the resonant frequency by up to 75%. In scenarios where the resonant frequency was higher than the notch frequency, the proposed filter successfully suppressed current oscillations, maintaining stability where conventional filters failed. The adaptive mechanism responded to instability within 2 ms, adjusting the notch frequency to optimal levels within 10 ms. This research contributes to the advancement of grid-connected power electronics, offering a robust solution that can enhance the reliability and efficiency of renewable energy integration by up to 30% under varying grid conditions.

Keywords: Adaptive digital notch filter, system stability, resonance mitigation, power electronics, renewable energy integration, single phase inverter grid.

1. Introduction

The rapid expansion of renewable energy sources has led to a significant increase in the number of renewable power generators connected to the electrical grid. This transition towards cleaner energy, while promising, has introduced new challenges in maintaining grid stability and power quality [1]. Grid-connected

inverters play a crucial role in integrating these renewable energy sources with the existing power infrastructure. However, when these inverters are disconnected from the grid due to shut-off operations, a sudden imbalance between power supply and demand can occur, resulting in voltage and frequency fluctuations that deteriorate overall system quality. This impact becomes more pronounced as the grid's reliance on renewable energy sources increases [2].

A particular concern arises with the widespread use of LCL (Inductor-Capacitor-Inductor) filters in grid-connected inverters. While these filters offer superior harmonic suppression compared to traditional L filters [3], they introduce resonance issues that can potentially destabilize the system. In extreme cases, fluctuations in system impedance can cause large-scale inverter disconnections, potentially resulting in widespread power outages [4]. Given these challenges, it is imperative that grid-connected inverters maintain stability and continue to supply energy reliably, even in the face of grid impedance fluctuations.

This paper proposes an adaptive digital notch filter for single-phase grid-tied inverters controlling inverter-side current. The proposed solution aims to maintain system stability even under fluctuating system impedance conditions by detecting resonance in real-time and rapidly stabilizing the system through dynamic adjustment of the notch frequency. The adaptive notch filter has been designed and tested through simulations, for a 3 kW single-phase grid-connected inverter, demonstrating its effectiveness in maintaining system stability under various operating conditions.

2. Related Research

Researchers have explored various stabilization techniques for LCL filter-based inverters, broadly categorized into passive damping and active damping approaches. Passive damping involves inserting a resistor into the LCL filter to reduce system resonance. While straightforward to design, this method has the disadvantage of increasing power losses in the system [5]. Active damping techniques, on the other hand, aim to stabilize the system without incurring additional losses.

Several active damping approaches have been proposed in the literature. The virtual resistance technique adds a capacitor current feedback loop to simulate the presence of a real resistor in the system. However, it requires additional sensors, which can increase system complexity and cost [6]. Another approach stabilizes the system through simple gain adjustments, taking into account the time delays inherent in digital controllers. While easy to implement, its application is limited by the ratio of resonant frequency to sampling frequency [7].

Notch filter-based techniques have also been explored, using notch filters to suppress system resonance by matching the notch frequency to the system's resonant frequency. These methods are relatively simple to design but can become ineffective when system parameters fluctuate [6], [7]. To address the limitations of fixed notch filters, researchers have proposed various techniques for real-time system impedance estimation, including methods based on Digital Fourier Transform (DFT) [8], [9] and Recursive Least Square (RLS) algorithms [10], [11]. However, these methods often suffer from limitations such as high computational complexity or the inability to operate during inverter operation.

The adaptive digital notch filter proposed in this paper builds upon and extends the work of these previous studies. By addressing the limitations of existing techniques, particularly in dealing with system parameter fluctuations, this study aims to contribute to the development of more robust and reliable grid-connected inverter systems for renewable energy applications. The proposed solution offers real-time detection of resonance caused by changes in system resonant frequency due to parameter fluctuations and provides rapid system stabilization by dynamically adjusting the notch frequency based on the direction of resonant frequency changes.

3. Methodology

A. Grid-connected Inverter System and Grid-connected System Model

The grid-connected inverter system under study (*Fig. 1*) consists of a grid-tied inverter coupled with an LCL filter. This configuration is crucial for harmonics suppression but introduces challenges in system stability, particularly when digital controllers are employed. The use of digital controllers, while offering precise control, introduces time delays that can significantly impact the system's stability, especially in the high-frequency domain. These delays, including ADC sampling, computation time, and PWM update periods, can have substantial effects on the high-frequency response characteristics. Consequently, when using LCL filters, it is imperative to account for these time delays to accurately determine system stability in the high-frequency region.

The system's signal processing can be conceptualized as a continuous block divided into continuous and digital subsystems (*Fig. 2*). The process begins with the feedback of the inverter current $I_i(s)$, which is converted to a digital signal $I_i(z)$ through an analog-to-digital converter (ADC). This digital signal is then compared with the reference signal $I_i^*(z)$, and the resulting error is processed by the controller $C(z)$. Finally, the control signal is converted back into a continuous signal via a Pulse Width Modulation (PWM) converter.

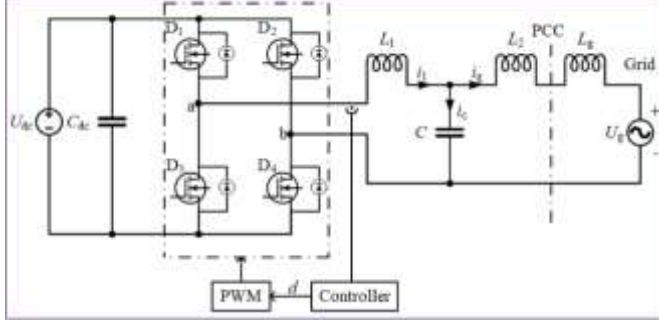


Figure 1: System diagram LCL filter, inverter and grid

In this model, the PWM converter effectively acts as a digital-to-analog converter (DAC), transforming the digital control signal into an analog form. The PWM process can be represented as a combination of a Zero-Order Hold (ZOH) and a total delay component e^{-sT_s} , where T_s is the sampling period. This delay accounts for the time required for ADC conversion, computation, and PWM generation within each sampling cycle.

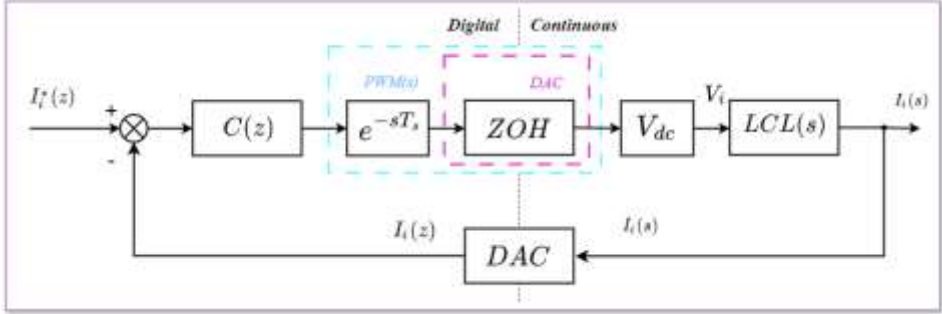


Figure 2: Control signal generation for inverter control

The controller $C(z)$ is implemented by converting two key components from the continuous to the discrete domain: a Proportional-Resonant (PR) controller $PR(s)$ and a notch filter $Notch(s)$. The transfer functions for these components are given by:

$$PR(s) = K_p + K_i \frac{2\omega_d s}{s^2 + 2\omega_d s + \omega_c^2}, \quad (1)$$

$$Notch(s) = \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}. \quad (2)$$

The PR controller is designed to track the fundamental grid frequency component, with zero steady-state error. The PR controller's resonant frequency

ω_c is typically set to match the grid frequency ω_g . This design allows for a significant gain increase at ω_c , effectively enabling precise current tracking at the grid frequency.

The notch filter is designed to greatly reduce the gain at its notch frequency ω_n . By aligning ω_n with the resonant frequency of the LCL filter, the filter can effectively cancel out the LCL filter's resonant peak, thereby stabilizing the system. The implementation of these controllers and filters in the digital domain requires careful consideration of the z-transform method. While the Tustin technique is commonly used in power electronics applications due to its excellent frequency characteristic reproduction, it can lead to frequency warping when applied to notch filters, especially when the notch frequency approaches the Nyquist frequency. To mitigate this issue, a pre-warping technique is employed for the notch filter conversion, ensuring accurate representation of the desired notch frequency in the digital domain.

The LCL filter's transfer function from inverter output voltage to line current, with the grid voltage V_g acting as a disturbance input, can be represented as a block diagram (Fig. 3). The grid voltage disturbance cannot be directly controlled and can adversely affect the inverter current I_i . The PR controller's high gain at the grid frequency effectively reduces this disturbance impact, simplifying system analysis and improving current quality.

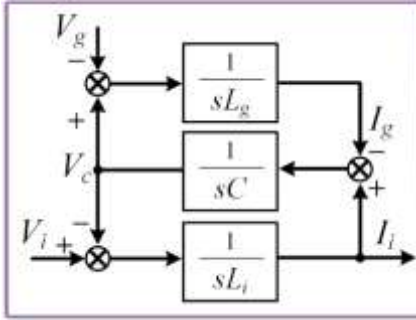


Figure 3: LCL Filter Block diagram

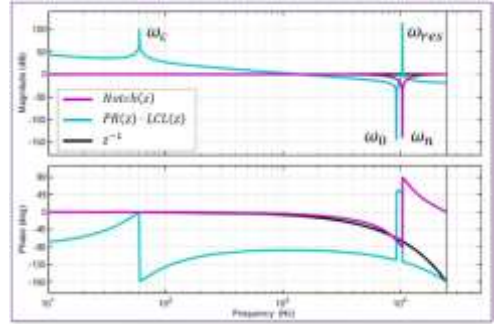


Figure 4: Bode diagram showing the frequency responses of a notch filter, Proportional-Resonant controller, and LCL filter

The transfer function $LCL(s)$ is expressed as:

$$LCL(s) \approx \frac{1}{L_1 L_2 C s^3 + (L_1 + L_2) s}. \quad (3)$$

The resonant frequency ω_{res} and the characteristic frequency ω_0 are expressed as:

$$\omega_{res} = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C}}, \quad (4)$$

$$\omega_0 = \frac{1}{\sqrt{L_1 C}}. \quad (5)$$

This comprehensive model of the grid-connected inverter system, incorporating the LCL filter dynamics, digital control effects, and the interaction between continuous and discrete domains, provides a solid foundation for analyzing system stability and designing effective control strategies. It allows for a nuanced understanding of the system's behavior across different frequency ranges and forms the basis for developing advanced control techniques, such as the adaptive digital notch filter proposed in this study.

Using the Tustin transformation with pre-warping for the notch filter, these transfer functions, from the equations (1), (2) and (3) are converted to their discrete-time equivalents used in the Bode plots:

$$PR(z) = K_p + K_i T_s \cdot \frac{z^2 - 1}{z^2 - 2\cos(\omega_c T_s)z + 1}, \quad (6)$$

$$Notch(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}, \quad (7)$$

where,

$$\begin{aligned} b_0 &= \frac{4 + \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}{4 + \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}, \quad b_1 = \frac{2\omega_n^2 T_s^2 - 8}{4 + \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}, \quad b_2 = \frac{4 - \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}{4 + \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}, \\ a_1 &= \frac{2\omega_n^2 T_s^2 - 8}{4 + \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}, \quad a_2 = \frac{4 - \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}{4 + \frac{2\omega_n T_s}{Q} + \omega_n^2 T_s^2}, \\ LCL(z) &= \frac{T_s^2}{L_1 L_2 C} \cdot \frac{(z+1)^2}{(z-1)^2} \cdot \frac{1}{z}. \end{aligned} \quad (8)$$

These discrete-time transfer functions are used to generate the Bode plots shown in *Fig. 4*, providing a complete frequency response analysis of the digital control system.

B. Analysis of the stability of a grid-connected inverter using a notch filter

The stability analysis of the grid-connected inverter system employing a notch filter is crucial for understanding the system's behavior under various conditions. The parameters used in this stability analysis, as well as in subsequent simulations are: $L_i = 330 \mu\text{H}$, grid inductance L_g ranging from 40-140 μH (corresponding to weak and strong grid conditions), $C=3 \mu\text{F}$, $V_{dc} = 380 \text{ V}$, $f_g = 50 \text{ Hz}$, $f_s = 50 \text{ KHz}$, with resulting ω_{res} range of 58,200-95,600 rad/s, and ω_n range of 16,000-90,000 rad/s. For maximum stability sensitivity, our analysis neglects the resistor component of the LCL filter, as smaller resistance values increase the likelihood of system instability.

Our analysis focuses on two key scenarios: when the system resonant frequency and the notch frequency coincide, and when they do not match.

Scenario 1: Matched Resonant and Notch Frequencies

The system stability can be determined through Bode diagram analysis when the resonant frequency matches the notch frequency. *Fig. 4* illustrates the Bode diagram for the notch filter $Notch(z)$, the PR controller $PR(z)$, the LCL filter $LCL(z)$, and the delay component z^{-1} with L_g at 100 μH and ω_n at 65,900 rad/s. For this scenario, the transfer function of the LCL filter from inverter voltage to line current in the discrete domain is:

$$LCL(z) = \frac{T_s^2}{L_1 L_2 C} \cdot \frac{(z+1)^2}{(z-1)^2} \cdot \frac{1}{z} \quad (9)$$

As shown in *Fig. 4*, the alignment of notch frequency ω_n with the resonant frequency ω_{res} results in resonant peak cancellation, ensuring system stability.

Scenario 2: Unmatched Resonant and Notch Frequencies

For analyzing stability when frequencies differ, we examine the resonant pole's path in the z-plane as the grid impedance varies. *Fig. 5(a)* shows this path with ω_n fixed at 65,900 rad/s while L_g increases from 40 to 140 μH , causing the frequency ω_{res} to decrease from 96,600 rad/s to 58,200 rad/s.

The resonant pole travels along path A in the z-plane, with instability occurring when L_g is less than 50 μH or greater than 100 μH , as the pole moves outside the unit circle.

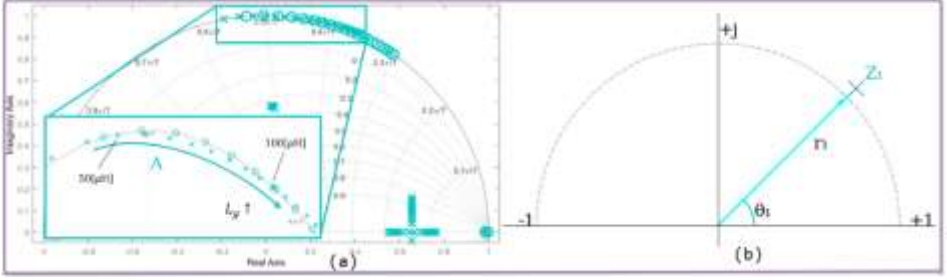


Figure 5: (a) Pole trajectory mapping with parametric variation of grid-side inductance (L_g : 40 $\mu\text{H} \rightarrow 140 \mu\text{H}$) at constant natural frequency ($\omega_n = 65,900 \text{ rad/s}$).
 (b) Complex plane representation showing a point z_1 on the unit circle

The resonant pole travels along path A in the z -plane. Critically, we observe that the system becomes unstable when L_g is less than 50 μH or greater than 100 μH , as the pole moves outside the unit circle under these conditions.

C. Proposed Adaptive Digital Notch Filter

The adaptive digital notch filter maintains system stability by dynamically adjusting its notch frequency based on real-time stability assessment and resonant frequency change detection through inverter-side current monitoring.

To understand the filter's operation, we first need to examine how the resonant pole affects the temporal response of the system. Fig. 5(b) illustrates this relationship in the z -plane. If we consider an arbitrary point z_1 on the z -plane, located at a distance r_1 from the origin and forming an angle θ_1 with the x -axis, we can express z_1 as:

$$z_1 = r_1 e^{j\theta_1} = e^{\sigma_1 + j\omega_1 T_s} . \quad (10)$$

According to the z -transform definition, the time response $C_1(t)$ corresponding to z_1 is:

$$C_1(t) = e^{\sigma_1 t} \sin(\omega_1 t + \phi) . \quad (11)$$

If $r_1 < 1$ (i.e., the pole lies within the unit circle), σ_1 becomes negative, indicating a stable system.

When the system becomes unstable, the resonant pole moves outside the unit circle, resulting in a positive σ_1 . This leads to the presence of a growing resonant frequency component in the output. Fig. 6 demonstrates this scenario, simulating the system's response when L_g increases from 100 μH to 140 μH at $\omega_n = 65,900 \text{ rad/s}$. The inverter-side current (Fig. 6(a)) and its Fourier spectrum (Fig. 6(b)) clearly show the increase in high-frequency components near the resonant frequency.

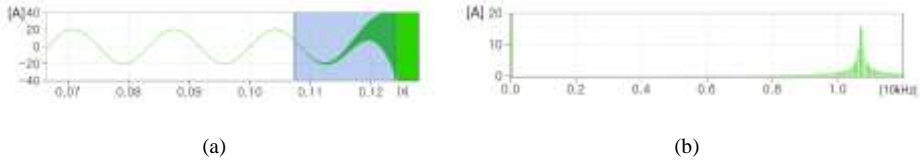


Figure 6: Unstable situation, (a) Current from inverter, (b) Fourier spectrum

The adaptive filter operates in three sequential steps:

1. Detect resonant condition occurrence.
2. Determine resonant frequency change direction.
3. Adjust notch frequency until resonant component amplitude decreases.

D. Resonance Detection

The resonance detection system monitors harmonic components in the inverter current and generates a resonance indicator (res_{ind}). The inverter-side current i_i consists of multiple components: the fundamental grid frequency component, switching frequency harmonics, and potential resonant frequency components. While switching harmonics exist at multiples of the switching frequency, they are typically of much lower magnitude than the fundamental and resonant components due to the LCL filter's attenuation characteristics. For the purpose of resonance detection, we focus on the dominant components in the error between reference i_i^* and actual current i_i :

$$\text{Error}(t) = i_i^* - i_i = A \sin(\omega_g t + \phi_g) + B \sin(\omega_{res} t + \phi_{res}), \quad (12)$$

where: ϕ_g represents the phase shift of the fundamental component due to control system delays, ϕ_{res} represents the phase shift at resonant frequency due to computational and sampling delays in the digital implementation. Higher-order harmonics are omitted from this expression as they don't significantly impact the resonance detection process.

Differentiating this error to separate high-frequency components:

$$\frac{d(\text{error})}{dt} = A \omega_g \cos(\omega_g t + \phi_g) + B \omega_{res} \cos(\omega_{res} t + \phi_{res}). \quad (13)$$

Given that the fundamental frequency is much smaller than the resonant frequency, and minimal computational delay ϕ_{res} is typically very small, we can approximate this as:

$$\frac{d(\text{error})}{dt} \approx B \omega_{res} \cos(\omega_{res} t + \phi_{res}). \quad (14)$$

The absolute value of this expression, when passed through a Low Pass Filter (LPF), provides the res_{ind} value, which represents the magnitude of the resonant frequency component. The resonance detection process is illustrated in the block diagram shown in Fig. 7. The derivative signal is passed through an absolute value function $|\cdot|$ to obtain the magnitude of the resonant component. Finally, the signal is low-pass filtered (LPF) to extract the resonance indicator res_{ind} , which represents the amplitude of the resonant frequency component in the current:

$$res_{ind} = LPF \left(\left| \frac{d}{dt} \text{error}(t) \right| \right). \quad (15)$$

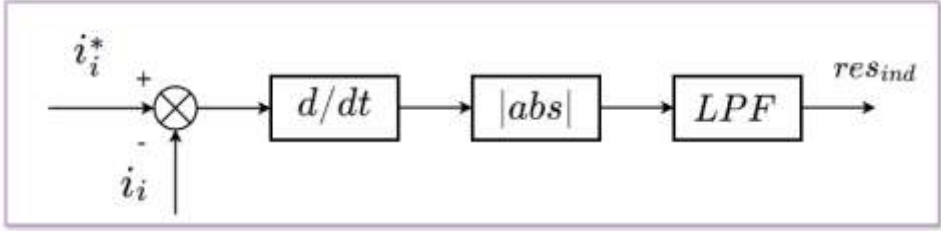


Figure 7: Resonance detection process

By monitoring the res_{ind} value, the system can detect the occurrence of resonance and the direction of change in the resonant frequency, as discussed earlier. This information is then used by the adaptive notch filter to dynamically adjust its frequency and maintain system stability.

E. Resonant Frequency Change Detection

To determine the resonant frequency's direction of change, the system slightly increases the notch frequency and observes the resulting change in the resonant component amplitude.

The resonant pole's behavior varies with notch frequency adjustments as shown in Fig. 5. With L_g at 100 μH and ω_n increasing from 15,900 rad/s to 85,900 rad/s, the pole follows a counterclockwise path.

For notch frequencies below the resonant frequency, increasing ω_n reduces oscillation amplitude. Conversely, for notch frequencies above the resonant frequency, increasing ω_n increases oscillation amplitude.

Therefore:

- Increased resonant component amplitude indicates decreased system resonant frequency;
- Decreased resonant component amplitude indicates increased system resonant frequency.

This adaptive mechanism maintains system stability across varying grid conditions by continuously adjusting the notch frequency based on these observations.

Through this comprehensive stability analysis, we gain valuable insights into the behavior of the grid-connected inverter system under various conditions. These findings inform the design of our adaptive digital notch filter, enabling it to respond effectively to changes in system parameters and maintain stability across a wide operating range.

4. Simulation results

The proposed adaptive digital notch filter was evaluated through simulation studies. The system parameters used for these evaluations were set within their operational ranges: ($L_i = 330 \mu\text{H}$, $L_g = 40 - 140 \mu\text{H}$, $C=3 \mu\text{F}$, $V_{dc} = 380\text{V}$, $f_g = 50 \text{ Hz}$, $f_s = 50 \text{ KHz}$, $\omega_{res} = 58,200 - 95,600 \text{ rad/s}$, $\omega_n = 16,000 - 90,000 \text{ rad/s}$). For the simulation studies, L_g was fixed at $100 \mu\text{H}$ while the notch frequency was varied to analyze system stability under different conditions.

A. Resonant Frequencies Lower than Notch Frequencies

Fig. 7 and Fig. 8 illustrate the simulated waveforms for scenarios where the resonant frequencies are lower than the notch frequencies.

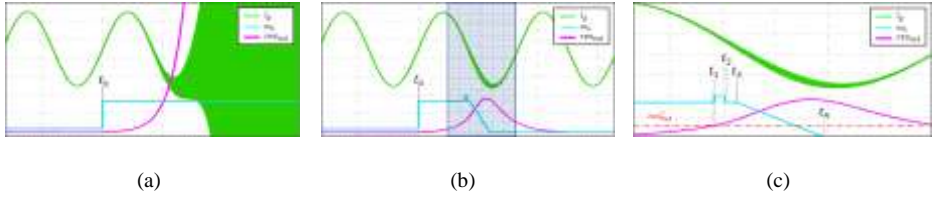


Figure 7: $\omega_n = 60,000 - 90,000 \text{ rad/s}$ at t_0 . (a) Conventional notch filter, (b) Proposed notch filter, (c) highlighted area in b

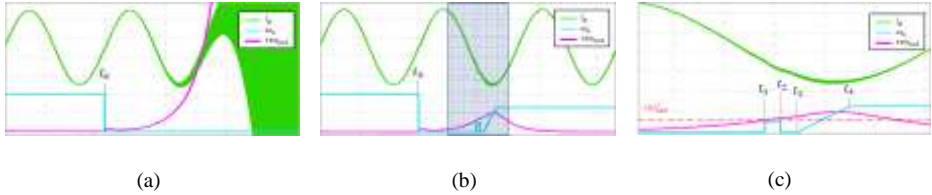


Figure 8: $\omega_n = 60,000 - 20,000 \text{ rad/s}$ at t_0 . (a) Conventional notch filter, (b) Proposed notch filter, (c) highlighted area in b

Fig. 7 demonstrates a simulated waveform using a conventional notch filter, where the notch frequency increases from 60,000 rad/s to 90,000 rad/s. This change causes the system to become unstable, resulting in current i divergence.

Fig. 7(b) shows the application of the proposed adaptive digital notch filter, with *Fig. 7(c)* providing a magnified view of the highlighted portion in *(b)*. The sequence of events proceeds as follows:

1. At t_1 : notch frequency increases, the system becomes unstable, causing the current harmonic magnitude to exceed the reference value res_{ind}^* , which triggers the adaptive mechanism.
2. From t_1 to t_2 : notch frequency is briefly raised to determine the direction of resonant frequency change.
3. From t_3 to t_4 : based on the negative slope of res_{ind} , the notch frequency is reduced, restabilizing the system.

B. Resonant Frequencies Higher than Notch Frequencies

Fig. 8 illustrates the simulated waveforms, for cases where the resonant frequencies exceed the notch frequencies.

Fig. 8(a) shows the system's response when the notch frequency decreases from 60,000 rad/s to 20,000 rad/s using a conventional notch filter. This reduction destabilizes the system, causing current i divergence.

Fig. 8(b) and *Fig. 8(c)* demonstrate the effectiveness of the proposed adaptive filter. The sequence of events is as follows:

1. At t_0 : when the notch frequency decreases, the system becomes unstable, causing the i harmonic magnitude to increase.
2. At t_1 : harmonic magnitude surpasses res_{ind}^* , activating the adaptive mechanism.
3. From t_1 to t_2 : notch frequency is increased, revealing that it's lower than the system resonant frequency.
4. At t_3 : notch frequency is further increased, successfully restabilizing the system.

These results demonstrate the robustness and effectiveness of the proposed adaptive digital notch filter in maintaining system stability across various operating conditions, outperforming conventional fixed-frequency notch filters.

5. Conclusion

This study presents an innovative adaptive digital notch filter designed to enhance the stability of grid-connected inverter systems employing LCL filters. The proposed solution addresses the challenges associated with varying grid

impedances and system resonant frequencies, which can lead to instability in conventional fixed-frequency notch filter designs.

Key findings and contributions of this research include:

1. *Comprehensive Stability Analysis*: We conducted a thorough analysis of system stability under various *conditions*, considering the interplay between the system resonant frequency and the notch frequency. This analysis provided crucial insights into the behavior of the resonant pole in the *z*-plane, informing the design of our adaptive filter.

2. *Adaptive Mechanism*: The proposed filter incorporates a three-step adaptive mechanism: the proposed filter incorporates a three-step adaptive mechanism: detection of resonance occurrence, determination of the direction of change in resonant frequency, and dynamic adjustment of the notch frequency.

3. *Improved Stability*: Simulation results demonstrate the superior performance of the adaptive digital notch filter compared to conventional fixed-frequency designs. The adaptive filter successfully maintains system stability across a wide range of operating conditions, including scenarios where the resonant frequency is both lower and higher than the initial notch frequency.

The adaptive digital notch filter presented in this study offers a significant advancement in the control of grid-connected inverter systems. By effectively mitigating the instability issues associated with LCL filters and varying grid impedances, this solution paves the way for more reliable and efficient grid-connected power electronics systems.

Future research directions may include:

- Extending the adaptive mechanism to handle multiple resonant frequencies;
- Investigating the performance of the adaptive filter in more complex grid scenarios, such as weak grids or micro-grids;
- Optimizing the adaptive algorithm for faster response times and reduced computational overhead.

In conclusion, the proposed adaptive digital notch filter represents a promising solution for enhancing the stability and performance of grid-connected inverter systems, contributing to the advancement of renewable energy integration and smart grid technologies.

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