Acta Universitatis Sapientiae
Electrical and Mechanical Engineering, 16 (2024) 1–15

DOI: 10.47745/auseme-2024-0001

# Compilation of Stress Concentration Factors in the Vicinity of a Geometric Discontinuity in Structures

Sahnoun ZENGAH[1], Abdelghani BALTACH[1,2], Abdelkader DJEBLI[1], Ali BENHAMENA[1]

[1] Mechanics of Materials, Energy and Environment Laboratory (L2M2E), University of Mascara, 29000 Mascara, Algeria,
e-mail: s.zengah@univ-mascara.dz
[2]Department of Mechanical Engineering, University of Tiaret, 14000, Tiaret, Algeria

Manuscript received September 19, 2024; revised November 06, 2024

**Abstract:** The analytical method estimates the stress concentration factor ($K_t$) using theoretical equations, while the numerical approach overcomes the limitations of analytical solutions in dealing with complex geometries. Simulations were conducted to evaluate how variations in hole diameter affect the stress concentration factor, revealing that larger diameters significantly increase the stress concentration factor. The presence of holes induces localized over-stress, compromising structural integrity. The research focuses on circular and elliptical holes under tensile loading, integrating both analytical and numerical techniques. Results validated against analytical solutions pinpoint areas of maximum stress concentration, which are crucial for the design and optimization of perforated structures. This study provides valuable insights for engineers, enhancing understanding of perforated plates' behavior under load, ultimately improving their reliability and safety in various industrial applications. The primary aim is to quantify the *SCF*, highlighting the risks posed by geometric discontinuities and their potential to lead to plastic deformation or structural failure.

**Keywords:** Stress concentration factors, Numerical analysis, FEM.

## 1. Introduction

Stress concentration is a prevalent issue in the mechanical design of components and parts. This phenomenon occurs as a localized increase in stress in regions where the geometry of the component is altered, typically at discontinuities or notches, often resulting from machining processes. The areas of stress concentration are usually where fatigue cracks initiate, and they can also lead to sudden failure in brittle materials. The phenomenon was first elucidated by G. Kirchhoff in 1898 [1], who analyzed the stresses around a hole.

Subsequently, numerous authors have developed analytical solutions for structures with progressively more intricate geometries [1, 2].

Some of the earliest rupture tests were conducted well before the Industrial Revolution, demonstrating that the tensile strength of iron wires varied inversely with their length. These findings suggested that material defects influenced strength [3]. These findings suggested that material defects influenced strength [4, 5]. This qualitative interpretation was later refined by Griffith in 1920, who established the theoretical foundation for fracture mechanics [1].

The utilization of ductile materials under tensile loads has, at times, resulted in failures occurring at stress levels well below the elastic limit. Initially, efforts to prevent these failures involved over-dimensioning structures; however, the need to lighten structures and reduce costs led to the emergence of fracture mechanics [4, 6].

Technically, evaluating and managing stress concentrations is essential to prevent premature structural failures. Finite element method (FEM) is commonly employed to model and predict stress levels in these critical zones. Such simulations help engineers identify high-stress areas and optimize designs to minimize stress gradients. To mitigate the adverse effects of stress concentrations, various approaches are considered. These include designing smooth transitions in geometries, implementing localized reinforcement with additional materials or inserts, and applying surface treatment techniques to smooth out critical zones. These measures aim to reduce stress levels and enhance material fatigue resistance.

Fracture mechanics was developed to investigate the propagation of macroscopic cracks. This approach is closely tied to the presence of discontinuities within materials, which alter stress, strain, and displacement states [3, 7]. Depending on the material and loading conditions, linear elastic fracture mechanics (LEFM) is utilized when plasticity is either absent or highly localized at the crack tip [8, 9].

### *1.1 Stress Intensity Factors*

Introduced by Irwin in 1957 [2], stress intensity factors correspond to specific kinematic movements of cracks. In the context of LEFM, stresses and strains near a crack exhibit an asymptotic expansion with a singular term expressed as:

$$\begin{cases} \sigma_{ij} = K_\alpha \dfrac{1}{\sqrt{2\pi r}} f_{ij}^{\alpha}(\theta) \\ \varepsilon_{ij} = K_\alpha \dfrac{1}{\sqrt{2\pi r}} g_{ij}^{\alpha}(\theta) \end{cases} \qquad (\alpha = I, II, III) \ . \tag{1}$$

The equation represents the singularity at $r = 0$ or asymptotic solution as tends to zero.

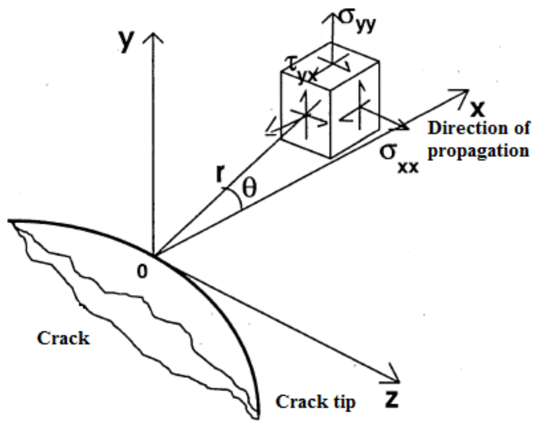


*Figure 1*: Crack in a polar coordinate system [10]

$K_\alpha$ is the magnitude of the singularity, it is called the Stress Intensity Factor (SIF) [10]. The stress intensity factor is given for the failure mode $\alpha$, with $\alpha$ being I, II, or III. The figure below (*Fig. 2*) represents the three failure modes.

The functions $f$ and $g$ provide the angular distribution, with their expressions being applicable to plane stress and strain conditions [11, 12].

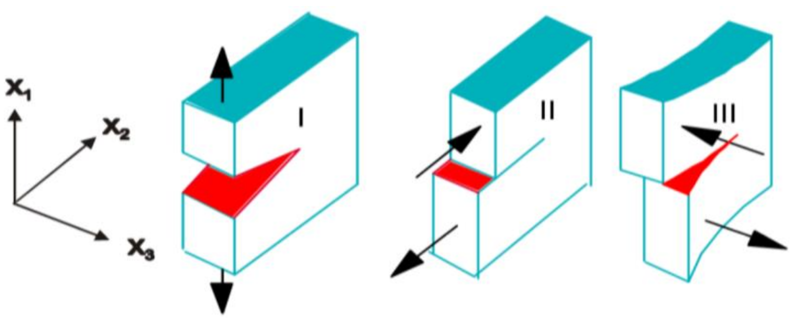


*Figure 2*: Modes of loading. (a) Cleavage mode: mode I.
(b) In-plane shear mode: mode II. (c) Antiplane shear mode: mode III. [11]

The system considered here us represented in *Fig. 3*. This later is an "infinite" panel containing a crack of length ($2a$) along the $x$ axis and subjected to uniform tension along $y$ axis.

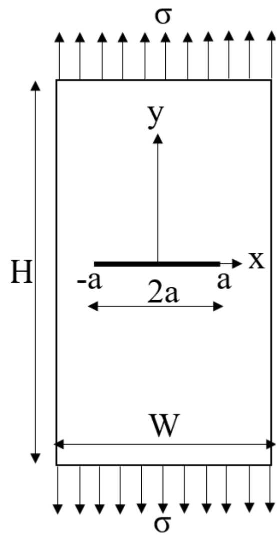


*Figure 3*: The center cracked plate in tension – CC(T) specimen [13]

In practice, such a model can be reasonably used when the dimensions of the crack are 10 to 20 times smaller than those of the plate [13, 14].

*1.2 Stress concentration phenomena*

Stress concentration phenomena are critically important in mechanics. Often unexpected and not always intuitive, they can lead to over-stress and, consequently, fatigue failures. Consider an elongated plate with width $W$ and thickness $t$ subjected to a tensile force $F$ [8, 15, 16, 17]. This plate experiences a stress defined by:

$$\sigma = F / W \cdot t \quad . \tag{2}$$

This represents the simplest case of uniaxial stress, serving as the primary definition of stress. Now, examining the same plate perpendicular to the tensile force, directly above the center of a hole, the thickness at that point is $(W – d)\ t$, where $d$ is the diameter of the hole:

$$\sigma = F / (W - d) \cdot t \quad . \tag{3}$$

This represents the force-to-section ratio, similar to the previous case. However, this is not accurate; at the edge of the hole, the stress is actually higher than this calculated value. Since static equilibrium dictates that the sum of stresses over the considered section must equal *F*, it follows that at other points, the stress must be lower than what is predicted by equation (3) [18].

### *1.3 Theoretical Stress Concentration Factor ($K_t$)*

Let us consider a thin plate having a elliptical hole of length 2a and subjected to uniaxial tension (see *Fig. 4*). The influence of a stress riser can be evaluated quantitatively through mathematical methods, including the stress concentration factor. This factor, denoted as $K_t$, is defined as the ratio of the maximum stress within the component to the reference stress, represented by the following equation:

$$K_t = 1 + 2 \cdot \frac{a}{b} = 1 + 2 \cdot \sqrt{\frac{a}{\rho}} \ , \tag{4}$$

where $a$ is the half-length of the notch and $\rho$ is the radius of curvature of the notch (with $a = \rho$). Consequently, $K_t$ can be approximated as $K_t = 3$:

$$K_t = \sigma / \sigma_{\text{nom}} \ . \tag{5}$$

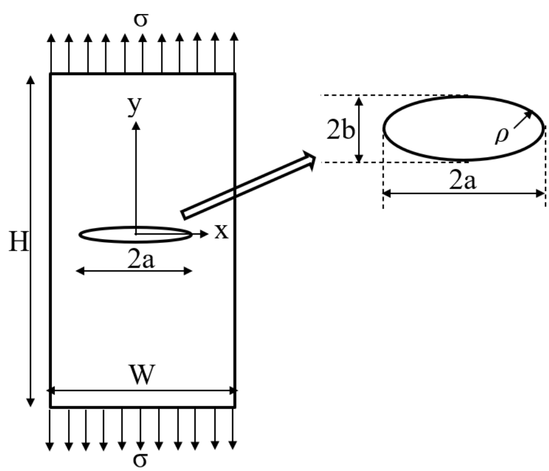


*Figure 4*: Infinite plate with elliptic hole under tension [19]

Furthermore, the stress concentration factor $K_t$ can be calculated from the following formula [20]:

$$K_t = \sigma_{\max} / \sigma \ , \tag{6}$$

where $K_t = \sigma_{max}/\sigma$ ; $\sigma_{max}$ is the maximum stress and $\sigma$ is the applied or global stress. The theoretical stress concentration factor $K_t$ depends solely on the geometry of the component and the type of loading.

## 2. Problem of a 3D Perforated Plate Under Tensile Stress Field

Consider a finite rectangular plate with dimensions h=100 mm, W=50 mm, and a thickness t=1 mm, containing a central circular hole of diameter d=12 mm. The plate is subjected to a uniform tensile load F=5 kN. The geometric configuration of the plate is illustrated in *Fig. 5*. The objective is to evaluate the stress concentration around the hole, both analytically using strength of materials principles and numerically through simulations conducted in ABAQUS.

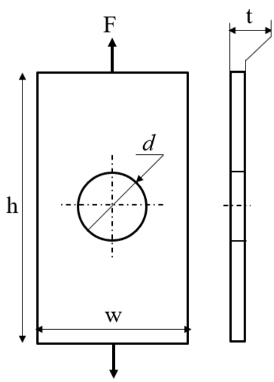


*Figure 5*: Perforated plate subjected to tensile stress

### *2.1. Analytical Study*

The purpose of this analysis is to evaluate the stress applied $\sigma_{app}$ in the section of the plate $S_1$, away from the hole:

$$S_1 = W \cdot t \ , \tag{7}$$

$$\sigma_{app} = \frac{F}{S_1} \ . \tag{8}$$

Calculating this gives $\sigma_{app} = 100$ MPa .

Next, it is necessary to calculate the nominal stress in the section passing through the center of the hole (refer to *Fig. 6*).

$$S_2 = (W - d) \cdot t \ , \tag{9}$$

$$\sigma_{nom} = \frac{F}{S_2} \ . \tag{10}$$

This yield $\sigma_{nom} = 131.57$ MPa .

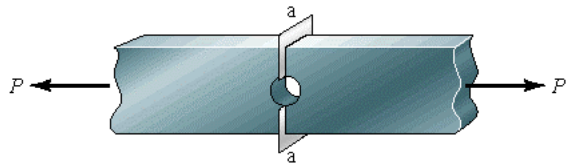


*Figure 6*: Section passing through the center of the hole

The maximum stress $\sigma_{\max}$ at the section intersecting the center of the hole (see *Fig. 7*) is determined using the following equation.

$$\sigma_{\max} = K_t \cdot \sigma_{nom} \ , \tag{11}$$

where: $\sigma_{\max}$ represents the maximum stress at the center of the hole, $K_t$ is the stress concentration factor, and $\sigma_{nom}$ denotes the stress at the center of the hole.


*Figure 7*: Maximum stress in the section passing through the center of the hole

The value of the stress concentration factor $K_t$ is obtained from graphical charts [21, 22]. In this study case:

$$\sigma_{\max} = K_t \cdot \sigma_{nom} \ , \tag{12}$$

$\frac{d}{W} = \frac{12}{50} = 0.24$; based on the charts, it can be concluded that $K_t = 2.4$; this leads to: $\sigma_{\max} = 315.76$ MPa.

### *2.2. Numerical Simulation of Stress Concentration Phenomenon*

The analytical resolution of mechanical problems is limited to specific cases. In contrast, numerical methods based on the discretization of these problems offer a very effective alternative and are commonly used in mechanics to solve complex situations. The finite element method (FEM) is the most widely used because it can handle problems with complex geometries and spans various fields of physics. Thanks to advancements in computing (processing power, visualization, and simulation tools), its implementation has become more accessible.

### *2.2.1. Geometric and Mechanical Characteristics*

The material used exhibits elastic behavior in accordance with Hooke's law.

*Table 1*: Mechanical and Geometric Characteristics of the Plate

| Young modulus *E* (MPa) | Poisson's Ratio $\nu$ | Length (mm) | Width (mm) | Thickness mm | Material | Loading $\sigma_{app}$ (MPa) |
|---|---|---|---|---|---|---|
| 270000 | 0.3 | 100 | 50 | 1 | Steel | 100 |

Representations of the geometry, the selected mesh, the boundary conditions, and the loading are provided in *Fig. 8*, *Fig. 9*, and *Fig. 10*.

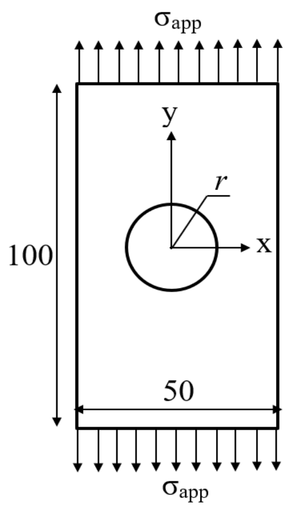


*Figure 8*: Geometry of the plate with circular hole

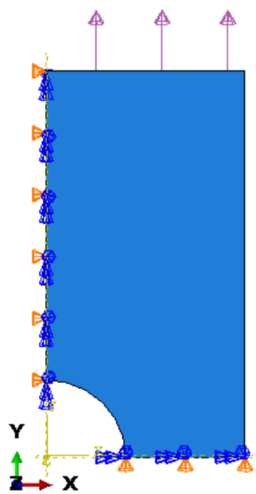


*Figure 9*: Boundary conditions and plate loading

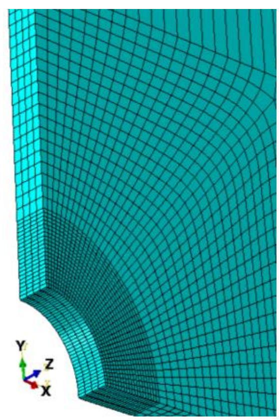

*Figure 10*: Mesh of the plate with circular hole

## 3. Results and Discussions

This section focuses on numerical simulations conducted using computational software (ABAQUS) to study and determine stress concentration in a plate subjected to a uniform tensile field. Particular attention is given to the influence of the hole diameter on the stress concentration factor $K_t$. Commonly used parameters are employed to analyze the behavior of perforated plates made from specific materials. The approach incorporates both analytical and numerical calculations of stress concentration, examining regions both at the edge and away from circular and elliptical holes under tensile loading. The presence of holes in structural components, commonly found in applications such as bolt holes or access ports, induces stress concentration, characterized by a localized increase in stress around the hole. The stress concentration factor (*SCF*) quantifies this increase and is influenced by the size of the hole. Larger holes generally lead to a higher *SCF*, as they disrupt the stress flow through the material more significantly, resulting in greater localized stresses. The *SCF* can be estimated using either analytical or numerical methods.

### *3.1. Plate with a central hole*

*Fig. 11* illustrates the distribution of the stress concentration factor, obtained from the results of the numerical simulation.

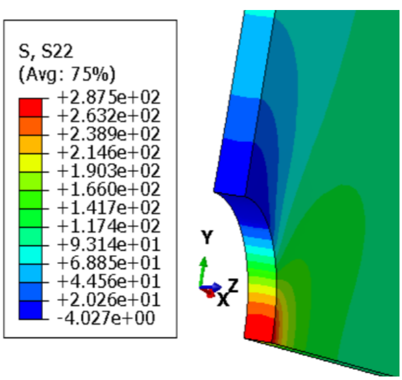


*Figure 11*: Circumferential stresses distribution at the edge of the circular hole ($r$ =10 mm case)

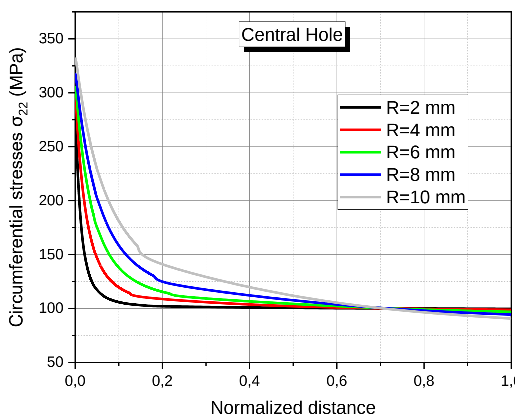


*Figure 12*: Variation of circumferential stresses as a function of normalized distance

*Fig. 11* and *Fig. 12* illustrate the variation of circumferential stresses under tensile loading as a function of normalized radius for various values of $r$ evaluated at specific intervals ($r$ = 2, $r$ = 4, $r$ = 6, $r$ = 8, $r$ = 10 mm).

*Fig. 13* illustrates how the Stress Concentration Factor (*SCF*) varies under tensile loading relative to the ratio $2 \cdot r/W$. This ratio allows for a comparative analysis of how the distance from the hole affects the stress concentration factor in relation to the plate width. Understanding these relationships is crucial for predicting failure locations and optimizing the design of perforated structures subjected to tensile stress.

The graph illustrates the variation in normal stress $\sigma$ as the radius $r$ increases from a reference point. Normal stress in tension represents the force per unit area, reflecting the material's response to stretching forces. The values of $r$ (2, 4, 6, 8, 10) likely indicate radial measurements from a specific point within the material or system under analysis. An increase in stress with radius may suggest a non-uniform stress distribution, whereas a decrease implies that the tensile effect diminishes with increasing distance from the point of applied force.

The *SCF* quantifies stress increase at a specific location compared to the average stress in the material, crucial for evaluating material performance and durability under load.

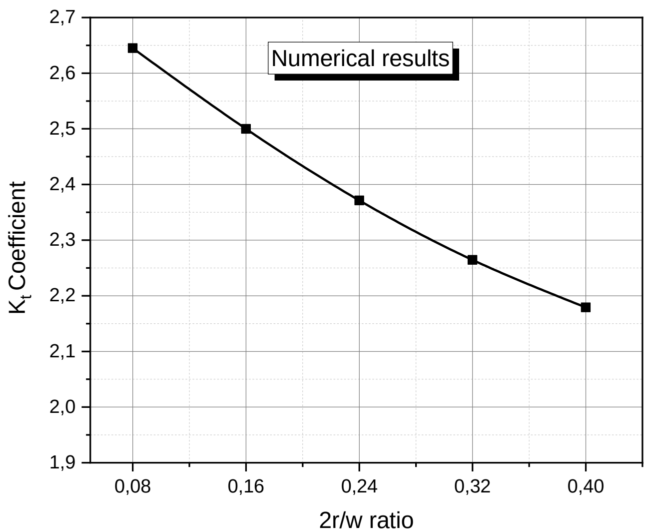


*Figure 13*: Variation of the $K_t$ of traction stress as a function of the $2 \cdot r/W$ ratio

### 3.1.1. Correlation and Validation

*Fig. 14* presents a comparison between the analytical results derived from relation and the numerical results. This comparison aims to validate the accuracy of the models used.

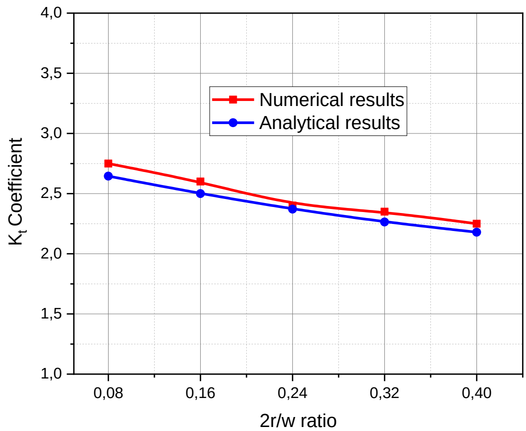


*Figure 14*: Comparison of the variation of the Stress Concentration Factor (*SCF*) under tensile loading between analytical and numerical results as a function of hole size $2 \cdot r/W$

### 3.2. Plate with a Half-Lateral Hole

Similar to case no. 1, the stress concentration in a plate with a half-lateral hole is analyzed. *Fig. 15* presents the distribution of the stress concentration factor, derived from the numerical simulation results.

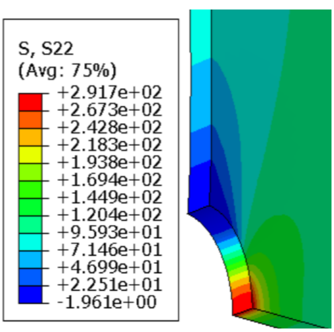


*Figure 15*: Circumferential stresses distribution at the edge of the circular hole

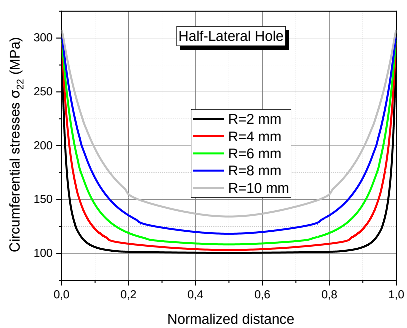


*Figure 16*: Variation of circumferential stresses as a function of normalized distance

*Fig 15* and *Fig. 16* illustrate the variation of normal stress under tensile loading as a function of distance. *Fig. 17* depicts the variations of the Stress Concentration Factor (*SCF*) under tensile loading relative to the ratio $2 \cdot r/W$. These figures provide critical insights into how the distance from the hole affects both the normal stress distribution and the *SCF*, which are essential for assessing the structural integrity of perforated components under tensile forces.

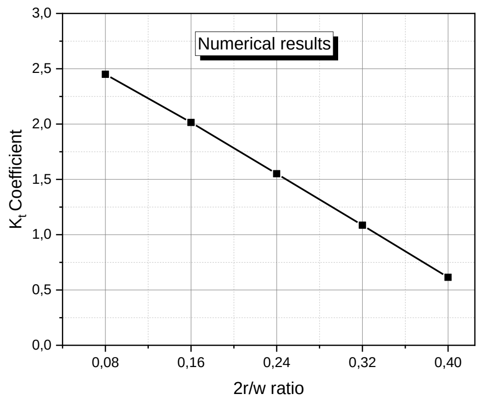


*Figure 17*: Variation of the *SCF* of traction stress as a function of the $2r/W$ ratio

### *3.2.1. Correlation and Validation*

For a clearer explanation, we present in *Fig. 18* the values of the stress concentration factor, calculated both analytically and numerically, as a function of the 2r/W ratio.

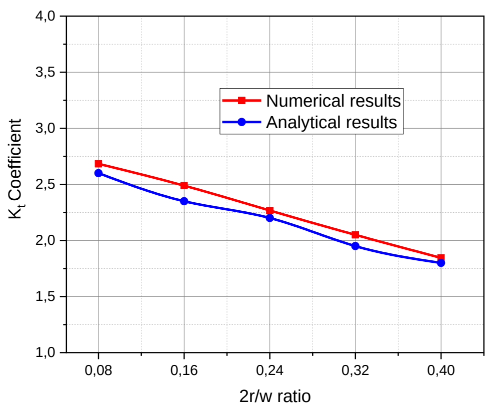


*Figure 18*: Comparison of the variation of the Stress Concentration Factor (*SCF*) under tensile loading between analytical and numerical results as a function of $2 \cdot r/W$

This figure illustrates the variation of the $K_t$ coefficient as a function of the $2 \cdot r/w$ ratio, where two sets of results are compared: numerical results and analytical results. The $K_t$ coefficient decreases monotonically as the $2 \cdot r/w$ ratio increases, indicating an inverse relationship between $K_t$ and $2 \cdot r/w$. The numerical and analytical results are very close throughout the curve, suggesting a strong agreement between the two methods.

### *3.3. Discussions*

Notably, the numerical simulation results for the first and second plates indicate that the Stress Concentration Factor *SCF* at the edge of the holes varies with changes in diameter. Specifically, the *SCF* value increases as the diameter enlarges. This trend highlights the importance of considering diameter variations in the design process to mitigate potential structural vulnerabilities. The analysis of the stress concentration factor in plates with holes indicates that this critical value increases proportionally with the hole diameter. This finding necessitates a careful approach to design and analysis to ensure optimal performance and adequate structural safety.

Particularly the Finite Element Method (FEM), is employed to model the mechanical behavior of structures with holes. These simulations allow for the prediction of stress and deformation patterns under various loading conditions and environmental factors. The FEM effectively captures complex geometries and boundary conditions that are often challenging to analyze analytically.

The Stress Concentration Factor *SCF* quantifies the localized amplification of stress around holes or other geometric discontinuities. This parameter is essential for evaluating the potential for structural failure, as high *SCF* values indicate areas where stresses may exceed material limits, leading to plastic deformation or fracture. Stresses are significantly higher near the edges of a hole due to the abrupt change in geometry, which concentrates stress in those regions.

A larger diameter results in a wider zone impacted by concentrated stresses, leading to higher *SCF* values as the diameter increases.

## 7. Conclusions

This study highlights the importance of considering geometric discontinuities, such as holes, in structural design. To address this, it investigates stress concentration in plates containing circular or elliptical holes under uniform stress fields, employing both analytical and numerical methods:

Analytical methods based on elasticity theory provide equations to estimate Stress Concentration Factors *SCF*, which are effective for simpler geometries. Finite Element Analysis (FEA) using ABAQUS software was also employed to simulate stress distributions around holes, offering advantages for complex geometries and loading conditions where analytical solutions are impractical. The software accurately models mechanical behavior under applied loads, predicting stresses and deformations.

The *SCF* increases with hole diameter, indicating higher stress levels at hole edges. The shape of the hole influences stress concentration differently; elliptical holes exhibit varying *SCF*s depending on orientation relative to the load. For instance, an ellipse with its major axis perpendicular to the load direction may have a higher *SCF* than one with its major axis parallel.

The ratio of hole diameter to plate width $d/W$ significantly impacts *SCF*. Proper hole sizing and placement are crucial to minimize stress concentrations and ensure structural integrity. Design optimization strategies may include adding local reinforcements or adjusting geometric shapes to distribute stress more evenly.

This study underscores the importance of considering geometric discontinuities like holes in structural design. Findings emphasize that *SCF* increases with hole diameter, necessitating careful design to avoid high stress concentrations.

For robust and reliable design solutions, engineers should integrate numerical simulations with experimental validation, ensuring structural performance and safety across various industrial applications.

## References

[1] Griffith, A., "The Phenomena of Rupture and Flow in Solids", *Philosophical transactions of the royal society of London. Series A, containing papers of a mathematical or physical character*, vol. 221, no. 582–593, pp. 163–198, 1920.

[2] Irwin, G.R., "Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate", *J. Appl. Mech*., vol. 24, pp. 361–364, 1957.

[3] Thomas, N., Weijermars, R., "Comprehensive atlas of stress trajectory patterns and stress magnitudes around cylindrical holes in rock bodies for geoscientific and geotechnical applications", *Earth-Science Reviews.*, vol. 179. pp. 303–371, 2018.

[4] Xue-feng, L., Kun, D., Jian, Z., Zhi-qiang, Y., "Failure behaviors of square-tunnel-structured rock specimens under various inclined boundary stress conditions", *Engineering Failure Analysis*, vol. 157, pp. 107888, 2024.

[5] Gibson, L.J., Ashby, M.F, "Cellular Solids: Structure and Properties", 2nd ed., Cambridge University Press, *1997.*

[6] Erdogan, F., "Fracture Mechanics", *Int. J. Solids and Structures.*, vol. 37, pp. 171–183, 2000.

[7] Ozkan, M.T., Erdemir, F., "Determination of stress concentration factors for shafts under tension", *Mater Test.,* Vol. 62, no. 4, pp. 413–421. 2020.

[8] Andia, P.C., Costanzo, F., Gray, G.L., "A Classical Mechanics Approach to the Determination of the Stress and Strain Response of Particle Systems. Model", *Simul. Mater. Sci. Eng*., vol. 14, pp. 741–757, 2006.

[9] Marcal, P.V., & King, I.P., "Elastic-plastic analysis of two-dimensional stress systems by the finite element method", *International Journal of Mechanical Sciences*, vol. 9, no. 3, pp. 143–155, 1967.

[10] Miannay, D., "Mécanique de la rupture", *Les Éditions de la Physique,* pp. 104-150, 1995.

[11] Raphaël, E., Gennes, P.G., "The adhesion between elastomers", *in Soft Order in Physical Systems. Boston, MA: Springer US,* pp. 57–71, 1994.

[12] Miao, X., Yu, Q., Zhou, C., Li, J., Wang, Y., & He, X., "Experimental and numerical investigation on fracture behavior of CTS specimen under I-II mixed mode loading", *European Journal of Mechanics - A/Solids,* vol. 72, pp. 235–244. 2018.

[13] Graba, M., "Catalogue of maximum crack opening stress for CC (T) specimen assuming large strain condition", *Open Engineering,* Vol. 3, no. 2, PP. 174–190, 2013.

[14] Mayr, C.M., Rother, K., "Improved stress concentration factors for circular shafts for uniaxial and combined loading", *Mater Test.*, Vol. 61, no. 3, pp. 193–203, 2019.

[15] Petersen, R.E., "Stress concentration factor", *ESDU Engineering data. Structures. London,* pp. 146–235 .1974.

[16] Santos, A., Guzman, R., Ramirez, Z., Cardenas, C., "Simulation of stress concentration factors in combined discontinuities on flat plates*". Journal of Physics: conference series IOP Publishing*, vol. 743, no 1, pp. 012014. 2016.

[17] Taylor, D., Kelly, A., Toso, M., Susmel, L., "The variable-radius notch: Two new methods for reducing stress concentration", *Eng.Fail. Anal*., vol. 18, pp. 1009–1017. 2011.

[18] Delyová, I., Hroncová, D., Sivák, P., "Use of FEM in identification of stress in place of concentration factor*", Am. J. Mech. Eng*., vol. 4, pp. 454–459. 2016.

[19] Tari, V., "Analytical verification of FFT-based micromechanical simulations near an elliptical crack tip by using composite voxels", *Mechanics of Advanced Materials and Structures* pp.1–10. 2023.

[20] Makhutov, N.A., Reznikov, D.O., "Generalization of Neuber's rule for the assessment of local stresses and strains in stress concentration zones for a wide range of applied strains", in *Proc. Procedia Struct. Integr,* 2019, pp. 199–206.

[21] Milella, P.P. "Fatigue and corrosion in metals", Springer Science & Business Media, 552. 16. 2012.

[22] Pilkey, W.D., Pilkey, D.F., Zhuming, B.I., "Peterson's stress concentration factors", Hoboken, John Wiley & Sons, 4th Edition E-Book ·January 2020.