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# On Some Pecularities of Paloid Bevel Gear Worm-Hobs

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**Abstract:** The paper discusses the generating surfaces of the paloid bevel worm hob used for paloid gear cutting. A pitch modification is required by this type of tool in order to ensure the optimal contact pattern by gearing. As a consequence of this modification the flank line of the plain gear tooth results as a paloid- a more general shaped curve than the theoretical involute of the basic circle. Equations of the generating surfaces result as equations of a generalized arhimedic bevel helical surface presenting those modifications that arise from the variation of the tooth thickness.

The first subsection discusses the essential geometrical peculiarities of the paloid worm hob. Here it is to remark that the most important characteristic of the tool is the variation of the tooth thickness on the rolling tape generator. The tooth thickness has its minimum value at the middle of the generator, and is maximum on the extremities. As a consequence, the generated gear tooth presents an opposite variation of thickness. The thickness variation is realized by moving the relieving tool on an ellipse, but this is not the only possible trajectory to be used.

The second subsection presents the generalized mathematical model of the tooth thickness variation. Starting from the ellipse used by classical relieving technologies, and writing the equation of the ellipse reported to the coordinate system of the paloid hob it results the radius function of the revolved surface of the reference helix. This function is used in its condensed form. With this, the developed mathematical model can be used for other forms of the relieving tool trajectory.

The next paragraphs present the matrix transformations between the coordinate systems of the hob and the relieving tool. Finally the parametric equations of the hob tooth flank are obtained. These equations depend on the radius modification function. Using other relieving trajectories that differ from an ellipse, other tooth flank forms will be obtained. Using this model, the flanks of the cut gear tooth can be easily written for all types of trajectories.

**Keywords:** paloid bevel worm hob, tooth thickness modification, generating surface.

## 1. General description

Paloid bevel gears are realized on Klingelnberg type teething machine-tools, using paloid bevel gear worm-hobs [1], [6]. These tools present a straight-shaped edge in their axial section. As a conclusion, the origin surface of the paloid worm hob is an Arhimedic bevel worm having the half taper angle of 30° as shown in *Fig. 1*. [4], [5]. The chip-collecting slots are axially driven. In order to realize the clearance angle on all edges, a helical relieving, perpendicularly oriented to the bevel generator is allowed.

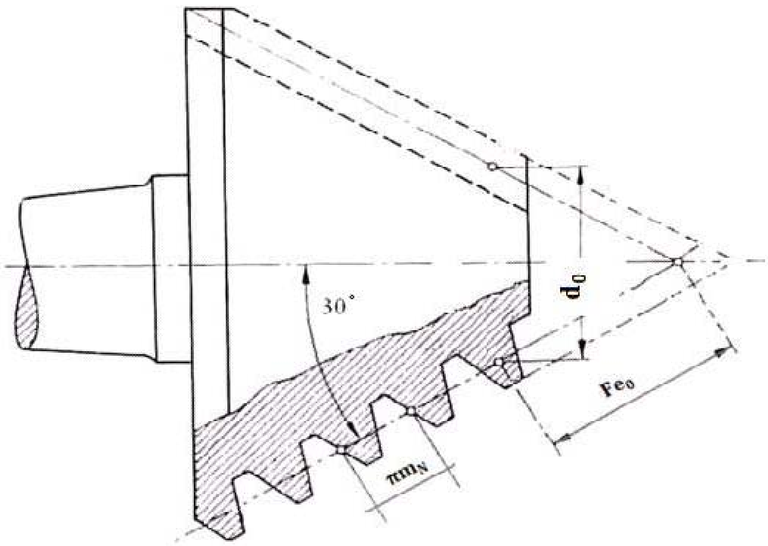


*Figure 1:* The paloid worm hob characteristic dimensions.

In order to ensure the good positioning of the contact patch by paloid gears, the pitch of the tool is variable but the tooth thickness (measured on the rolling taper generator) must increase at the extremities of the tool. Using this principle, the tooth thickness on the rolling tape generator increases in both directions starting from the point *N* (*Fig. 2*).

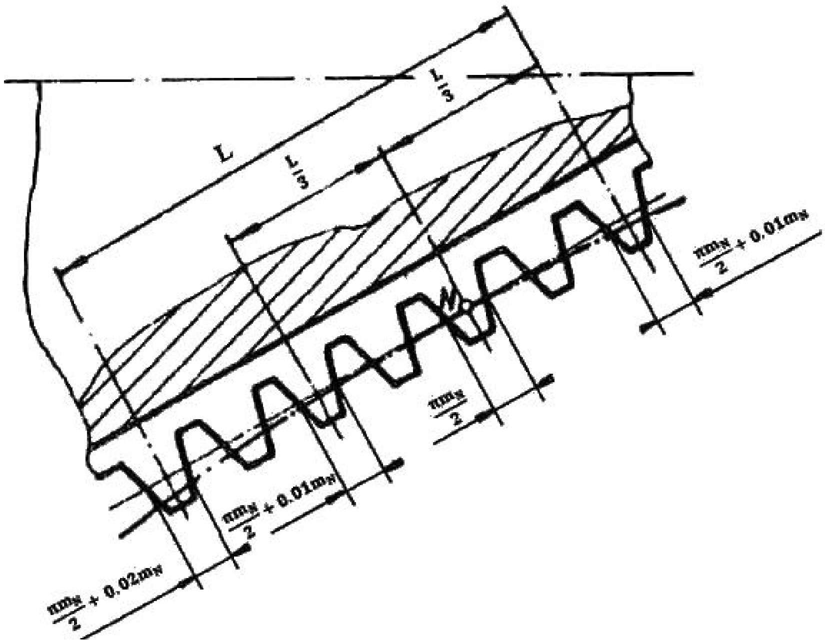


*Figure 2:* The tooth thickness distribution along the cone generator.

The thickness of teeth in case of gears manufactured using the worm-hob described above results smaller at the tooth extremities and larger in the middle of it. This localizes the contact pattern in the middle of the tooth-flank when gearing. This modification causes the deformation of the tooth line on the plain gear too, transforming the theoretical involute into a more general shaped curve named paloid.

Both the threading and the relieving operation are realized on a relieving machine, where the axis of the worm-hob is declined with a 30° angle reported to the axis of the chuck, in a horizontal plain containing the axis. This way the taper generator becomes parallel with the direction of the longitudinal slider.

The curved generator of the worm-hob is realized through leading the cutter slider by a profiled turning-template. The same template is used by the threading and relieving operations.

Bevel worm hobs present one thread, and allow the cutting of any teeth number by a fixed module and pressure angle. The pitch is normalized along the rolling cone generator and can be calculated using the formula: $p_N = \pi \cdot m_N$. The axial pitch (defined on the worm hob's axis) is determined by $p_A = \pi \cdot m_N \cdot \cos 30^\circ$.

## 2. The geometry of the tooth thickness modification

Considering the bevel worm-hob as a helical bevel surface, like any bevel thread, it is to mention that next to the axial pitch, a radial pitch can be defined, depending on the axial pitch and the half taper angle of the rolling cone (30°). If the generator of the worm hob were a straight line, the radial pitch value would satisfy the expression $h = p_A \cdot \mathrm{tg}30^o$. However, the generator is curved and the radial pitch is calculated taking into account the fact that the endpoint of the characteristic radius must be on that generator. Considering the tool's pressure angle $\alpha$, the distance between the curved and the straight generators (based on the dimensions of the tool profile indicated in [1], [2], [6]) can be calculated using the formula

$$\Delta_1 = (p_N + 0{,}01 \cdot m_N - p_N)/2 \cdot \mathrm{tg}\alpha = 0{,}01 \cdot m_N / 2 \cdot \mathrm{tg}\alpha$$

applicable for point *A* (*Fig 3*). The same distance is indicated related to point *C* (*Fig 3*). In point *D*, the distance discussed above will increase to double as follows from the formula $\Delta_2 = (p_N + 0{,}02 \cdot m_N - p_N)/2 \cdot \mathrm{tg}\alpha = 2\Delta_1$.

The curved generator is usually an arc of ellipse of which major axis (superposed to the *Ox*) is parallel to the rolling cone generator, and it's minor axis (superposed to *Oy*) passes through point *B* (*Fig 3*), situated at a distance of $F_{eo} + 2p_N$ from the top of the rolling cone.

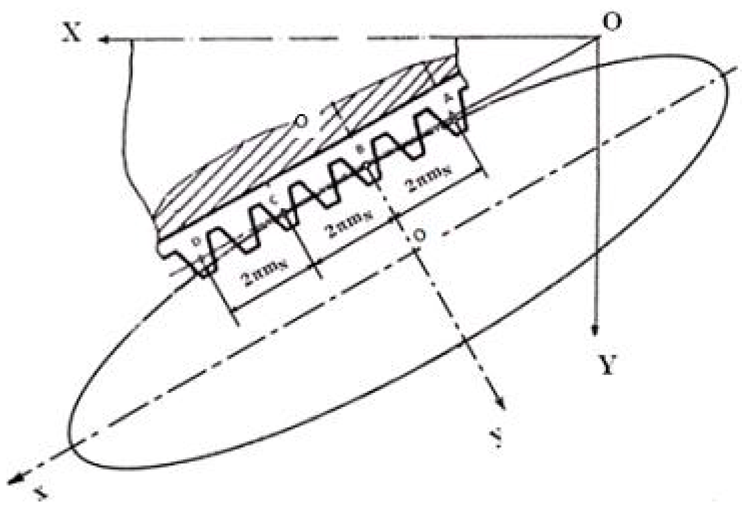


*Figure 3:* The elliptically curved generator.

It is to remark that the ellipse passes through the following points: $A(-p_N, a-\Delta_1)$; $B(0,a)$; $C(-2.p_N, \Delta_1)$ and $D(-4.p_N, a-\Delta_2)$.

The canonic equation of an arbitrarily shaped ellipse, reported to its axes, is:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0 \tag{1}$$

Writing the equation of the ellipses passing through the points *A*, *B*, *C* and *D* a system of equation will result, of which solution leads to the reference radius endpoint function e.g. $h = f(p_N / \cos 30^o)$. In this paper the radial pitch is accepted having a general form as expressed by the above formula. As a consequence, the relations deduced in the following can be used for other forms of the curved generator too. Using this form, the essential aspects of the dependences will not be influenced by the very sophisticated expressions of the radial pitch [3].

The matrix equation of the bevel worm hob's edge, reported to the self coordinate system, is:

$$r_M = \left\| \begin{matrix} \left(F_{eo} - \dfrac{\pi \cdot m_N}{4}\right)[\cos 30° + \mathrm{tg}30° \sin(30° - \alpha)] + \lambda \sin(30° - \alpha)] \\ \lambda \cos\alpha \\ 0 \\ 1 \end{matrix} \right\| \tag{2}$$

This edge is fixed to the mobile coordinate system $O_M X_M Y_M Z_M$ superposed at the initial moment with the fixed coordinate system $OXYZ$. The parametric form of the edge expressed in the mobile system will is denoted by $r_M$ but it keeps the form of $r_t$ given by (2).

The surface of the generator flank of the bevel worm hob can be obtained considering the followings: the edge is fixed to the mobile coordinate system (*Fig 4*) that executes a helical motion reported to the stationary system. The origin moves in the direction of axis with an amount of $p_A \cdot \varphi$, simultaneously with a rotation by angle $\varphi$ of the worm-hob (representing the workpiece in the above case).

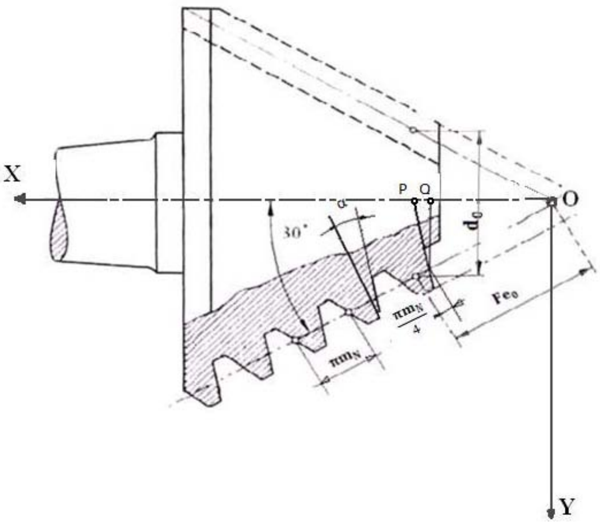


*Figure 4:* The position of the right edge related to the used reference system.

First the edge must be reported to an auxiliary coordinate system. Between this and the auxiliary system $O_M X_M Y_M Z_M$ there exist only translations by amounts of $p_A \cdot \varphi / 2\pi$ and $h \cdot \varphi / 2\pi$ respectively. A transfer matrix describing the above translations is:

$$M_{aM} = \begin{Vmatrix} 1 & 0 & 0 & \dfrac{p_A}{2\pi}\varphi \\ 0 & 1 & 0 & \dfrac{h}{2\pi}\varphi \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix} \qquad (3)$$

After that the edge's equation is transformed to the stationary system, mentioning that auxiliary system executes only a rotation around its *X* axis, characterized by the matrix

$$M_{Oa} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix} \qquad (4)$$

The pointing vector of the bevel helical surface described by the edge, reported to the stationary reference system is obtained from the following matrix equation

$$r = M_{Oa} \cdot M_{aM.} \cdot r_M = M_{OM} \cdot r_M \tag{5}$$

Multiplying $M_{Oa}$ by $M_{aM}$ it results

$$M_{OM} = \begin{Vmatrix} 1 & 0 & 0 & \frac{p_A}{2\pi}\varphi \\ 0 & \cos\varphi & -\sin\varphi & \frac{h}{2\pi}\varphi.\cos\varphi \\ 0 & \sin\varphi & \cos\varphi & \frac{h}{2\pi}\varphi.\sin\varphi \\ 0 & 0 & 0 & 1 \end{Vmatrix} \tag{6}$$

Finally, the matrix expression of the pointing vector is:

$$\begin{Vmatrix} X \\ Y \\ Z \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1 & 0 & 0 & \frac{p_A}{2\pi}\varphi \\ 0 & \cos\varphi & -\sin\varphi & \frac{h}{2\pi}\varphi.\cos\varphi \\ 0 & \sin\varphi & \cos\varphi & \frac{h}{2\pi}\varphi.\sin\varphi \\ 0 & 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} X_M \\ Y_M \\ Z_M \\ 1 \end{Vmatrix} \tag{7}$$

Expression (7) realizes the matrix form of the right flank of the bevel worm-hob tooth, reported to the stationary system *OXYZ*, attached to the hob.

Analytical expression is obtained after multiplying the matrices in the equation before. Similarly results the equations of the opposite flank, if starting the calculus with the equations of the opposite edge.

## References


[1] Krumme, W., “Klingelnberg-Spiralkegelräder Dritte neubearbeitete Auflage”, Springer Verlag, Berlin, 1967.

[2] *** “MASINOSTROIENIE”- ENCYCLOPEDY, Vol.VII., Masghiz Moskou.

[3] Máté, M., Hollanda, D. “The Enveloping Surfaces of the Paloid Mill Cutter”, in *Proc. 18th International Conference on Mechanical Engineering*, Baia Mare, 23-25 April 2010, pp. 291-294.

[4] Michalski, J., Skoczylas, L. “Modeling the tooth flanks of hobbed gears in the CAD environment", *The International Journal of Advanced Manufacturing Technology*, vol. 36, no. 7-8, 2008.

[5] Shu-han Chen, Hong –zhi Jan, Xing-zu Ming, “Analysis and modeling of error of spiral bevel gear grinder based on multi-body system theory", *Journal of Central South University of Technology*, vol. 15, no. 5, 2008.

[6] Klingelnberg, J. “Kegelräder- Grundlagen, Anwendungen”, *Springer Verlag*, 2008.