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Nonlinear Filtering in ECG Signal Denoising

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Abstract: This paper presents a non-linear filtering method based on the multiresolution analysis of the Discrete Wavelet Transform (DWT). The main idea is to use the time-frequency localization properties of the wavelet decomposition. The proposed algorithm is using an extra decomposition of the identified noise in order to reduce the correlation between the electrocardiogram (ECG) signal and noise. The linear denoising approach assumes that the noise can be found within certain scales, for example, at the finest scales when the coarsest scales are assumed to be noise-free. The non-linear thresholding approach involves discarding the details exceeding a certain limit. This approach assumes that every wavelet coefficient contains noise which is distributed over all scales. The non-linear filter thresholds the wavelet coefficients and subtracts the correlated noise. The used threshold depends on the noise level in each of the frequency bands associated to the wavelet decomposition. The proposed non-linear filter acts by thresholding the detail coefficients in a particular way, in order to eliminate the correlation between the noise and the signal. In this paper, in order to evaluate the proposed filtering method, signals from the MIT-BIH database have been used, and the filtering procedure was performed with added Gaussian noise. The proposed procedure was compared with ordinary wavelet transform and wavelet packet transform based denoising procedures, the followed parameters are the signal to noise ratio and the denoising error.

Keywords: Wavelet decomposition, wavelet shrinkage, non-linear filtering.

1. Introduction

The interest in using the wavelet transform to denoise the electrocardiogram (ECG) signals is increasing. The wavelet transform is a useful tool from time–frequency domain, preferred for the analysis of complex signals. The application of this transform to ECG signal processing has been found particularly useful due to its localization in time and frequency domains. The discrete wavelet transform- based approach produces a dyadic decomposition structure of the signals. In this way, the wavelet packet approach is an adaptive method using an optimization of the best tree decomposition structure independently for each signal.

2. Methods

The continuous wavelet transform (CWT) of the signal $x(t)$ is defined as a convolution of a the signal with a scaled and translated version of a base wavelet function, [1]:

$$W_a x(b) = \int_{-\infty}^{+\infty} x(t)\cdot\psi_{a,b}(t)\,dt = \frac{1}{\sqrt{|a|}}\int_{-\infty}^{+\infty} x(t)\cdot\psi\left(\frac{t-b}{a}\right)dt \tag{1}$$

where the scale '*a*' and translation '*b*' parameters are nonzero real values and the wavelet function is also real. A small value of '*a*' gives a contracted version of the mother wavelet function and then allows the analysis of high frequency components. A large value of the scaling factor stretches the basic function and provides the analysis of low-frequency components of the signal.

The discrete wavelet transform (DWT) is defined as a convolution between the analyzed signal and discrete dilation and translation of a discrete wavelet function. In its most common form, the DWT applies a dyadic grid (integer power of 2 scaling with '*s*' and '*l*') and orthonormal wavelet basis function: .

$$\psi_{(s,l)}(x) = 2^{-\frac{s}{2}}\psi\left(2^{-s}x - l\right) \tag{2}$$

The variables *s* and *l* are integers that scale and translate the mother function ψ to generate wavelets (analyzing functions). The scale index *s* indicates the wavelet's width, and the location index *l* gives its position. The mother wavelets are rescaled, or "dilated" by powers of two, and translated by integer '*l*' values. In this case we have a dyadic decomposition structure. These functions define an orthogonal basis, the so-called wavelet basis [3], [5]. The Discrete Wavelet Transform (DWT) decomposition of the signal into different frequency bands (according to Mallat's algorithm [2]) can be performed by successive high-pass and low-pass filtering (digital FIR structures) in the time domain followed by downsampling to eliminate the redundancy, as shown in *Fig.1*.

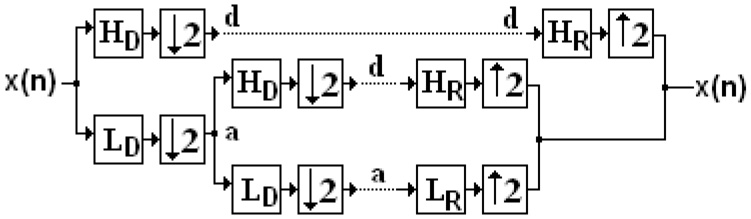

Figure: Second order dyadic scale decomposition and reconstruction.

In discrete wavelet analysis of $x(t)$ is decomposed on different scales, as follows:

$$x(t) = \sum_{j=1}^{K} \sum_{k=-\infty}^{\infty} d_j(k)\psi_{j,k}(t) + \sum_{k=-\infty}^{\infty} a_K(k)\varphi_{K,k}(t), \tag{3}$$

where $\psi_{j,k}(t)$ are discrete analysis wavelets and $\varphi_{K,k}(t)$ are discrete scaling functions, $d_j(k)$ are the detailed wavelet coefficients at scale 2^j and $a_K(k)$ are the approximated scaling coefficients at scale 2^K. The discrete wavelet transform can be implemented by the wavelet and scaling filters:

$$L(n) = \frac{1}{\sqrt{2}} \langle \varphi(t), \varphi(2t-n) \rangle, \tag{4}$$

$$H(n) = \frac{1}{\sqrt{2}} \langle \psi(t), \varphi(2t-n) \rangle = (-1)^n L(n), \tag{5}$$

which are quadratic mirror filters [3]. The estimation of the detail signal at level j will be obtained by convolving the approximation signal at level j-1 with the coefficients $L(n)$. Convolving the approximation coefficients at level j-1 with the coefficients $H(n)$ gives an estimate for the approximation signal at level j. This results in a logarithmic set of bandwidths. *Fig.2* shows the wavelet decomposition tree, the time-frequency blocks and the resulted bandwidths for a third order DWT decomposition. The first stage divides the spectrum into two equal parts. The second stage divides the lower part in quarters and so on. This results in a logarithmic set of bandwidths.

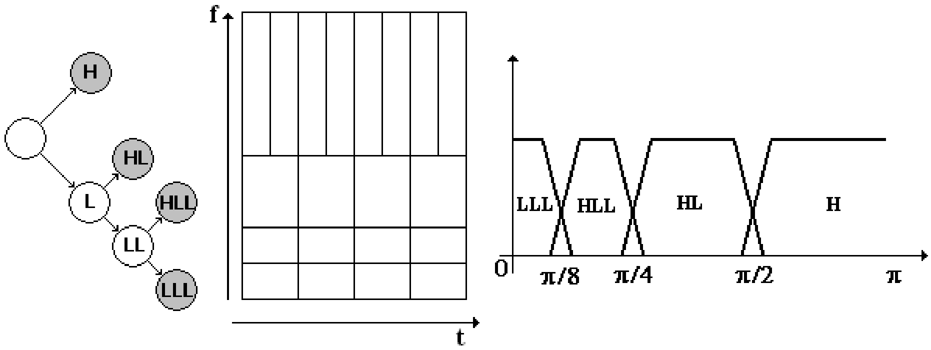

Figure 2: Time-frequency blocks and resulted relative bandwidths for third order dyadic scale decomposition.

The wavelet packet analysis is a generalization of discrete wavelet analysis providing a redundant decomposition procedure, where both detail and approximation signals are split at each level into finer components. This produces a decomposition tree as shown in *Fig. 3*.

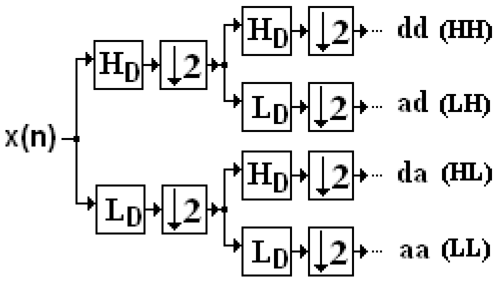

Figure 3. Wavelet packet decomposition tree.

The tree contains several admissible bases one of which is the wavelet basis itself. Having a large but finite library of bases it is possible to extract the best basis relatively to some criterion. The best basis algorithm finds a set of wavelet bases that provide the most desirable representation of the data relative to a particular cost function which may be chosen to fit a particular application [7]. This basis can be any subtree of the initial entire tree. The reconstruction procedure is similar to the inverse wavelet transform.

Figure 4 shows the time-frequency blocks for a second order wavelet packet decomposition, L and H are the low- and highpass filters with downsamplers.

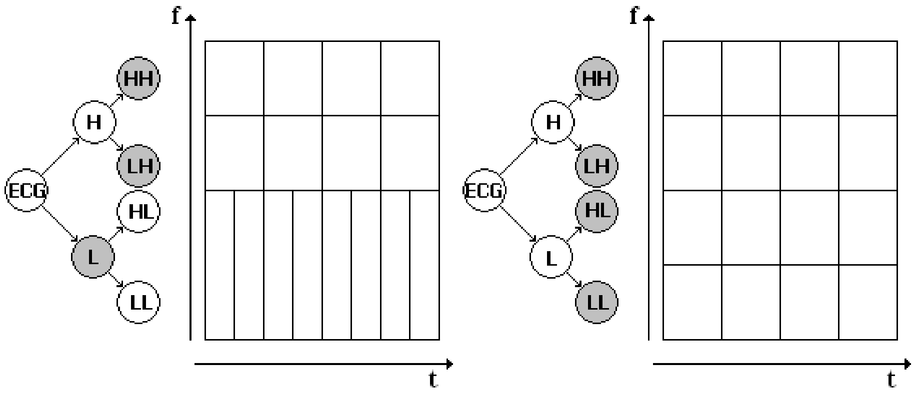

Figure 4: Time-frequency blocks for second order dyadic wavelet packet decomposition.

3. The proposed non-linear filtering method

The goal of any denoising procedure is to extract the useful signal from the noisy one, by eliminating the identified noise. The most simple model of the noisy signal is the superposition of the signal and a Gaussian type of noise. The main idea of non-linear filtering is to use the time-frequency localization properties of the discrete wavelet decomposition. The non-linear denoising approach assumes that every wavelet coefficient contains noise and it is distributed over all scales. The non-linear thresholding means discarding the detail coefficients exceeding a certain limit. There are two types of thresholding, the soft and the hard methods. With hard thresholding the coefficients which are lower than the threshold are set to zero. In soft thresholding, the remaining non-zero coefficients are shrunk toward zero. In this paper we assume that the identified noise is contained by the first order detail coefficients. It can be observed on the first order decomposition of an electrocardiogram signal that the first order detail coefficients shows some correlation with some characteristic points of the signal, like local maxima as shown in *Fig. 5*. The main idea in this work is to reduce this correlation by an extra decomposition followed by a nonlinear thresholding.

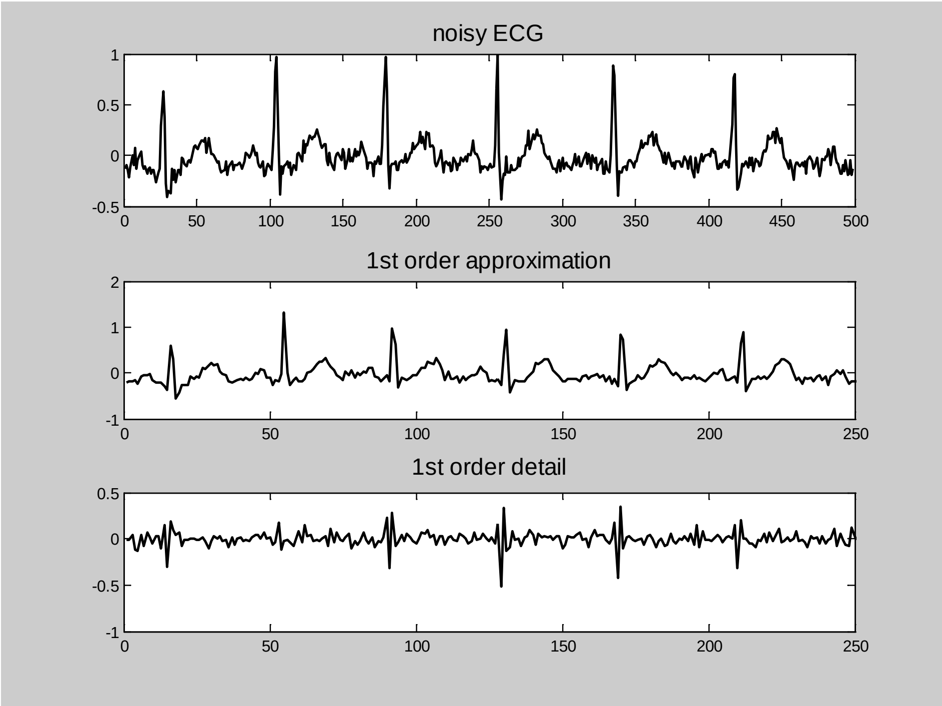

Figure 5: Correlation between first order detail and approximation coefficients.

The proposed non-linear filter acts by thresholding the detail coefficients in a particular way. The estimated noise, n_e, assumed to be the first order detail part is decomposed in a discrete wavelet structure, after that it is reconstructed only from the second order detail coefficients and is subtracted from the initial noise. The proposed smoothing procedure consists of:

1. third level DWT decomposition of the noisy ECG, resulting 3 detail coefficient sets (H, HL, HLL,) and an approximation coefficient set LLL;
2. two step second level decomposition of the estimated noise (H), resulting HHH, LHH, LH;
3. reconstruction of H' only from HHH, LHH;
4. the detail coefficients HL, HLL are thresholded;
5. reconstruction of the filtered signal through a third order Inverse Wavelet Transformation.

The procedure can be seen in *Fig. 6*. Only the detail coefficients were thresholded, the estimated correlation between noise and signal (LH) was removed. Thresholding the average coefficients (LLL) can lead to another type of filtering and can be the subject of another research paper. Both of hard and soft thresholds have been applied [4], [6].

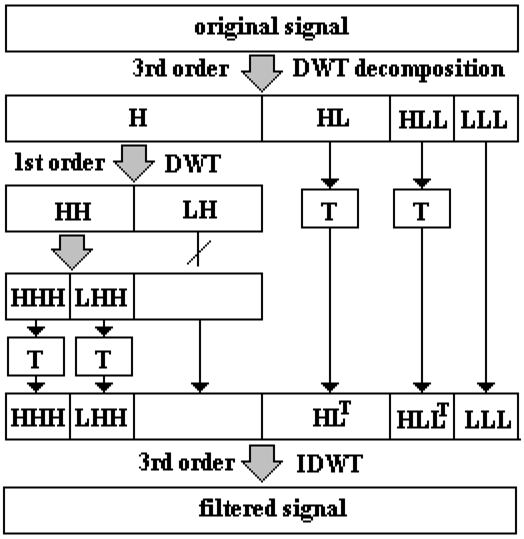

Figure 6: The proposed non-linear filtering procedure.

4. Results

To estimate the ability of this denoising procedure the followed parameters were the obtained signal to noise ratio and the absolute value of the error defined as:

$$SNR_1[dB] = 10\lg\left(\frac{PoriginalECG}{PoriginalECG - PdenoisedECG}\right) \tag{6}$$

$$Error = abs(originalECG - denoisedECG) \tag{7}$$

Figure 7 presents the original signal, the wavelet transform based denoised (soft thresholding) signal and the result of proposed nonlinear filtering. One can see (visual analysis) that the new method preserves more accurate information about signals characteristic points than the DWT based procedure. *Figure 8* shows the obtained signal-to-noise ratios by different denoising methods. The results show that the proposed method performs better denoising if the signal has lower signal-to-noise ratio.

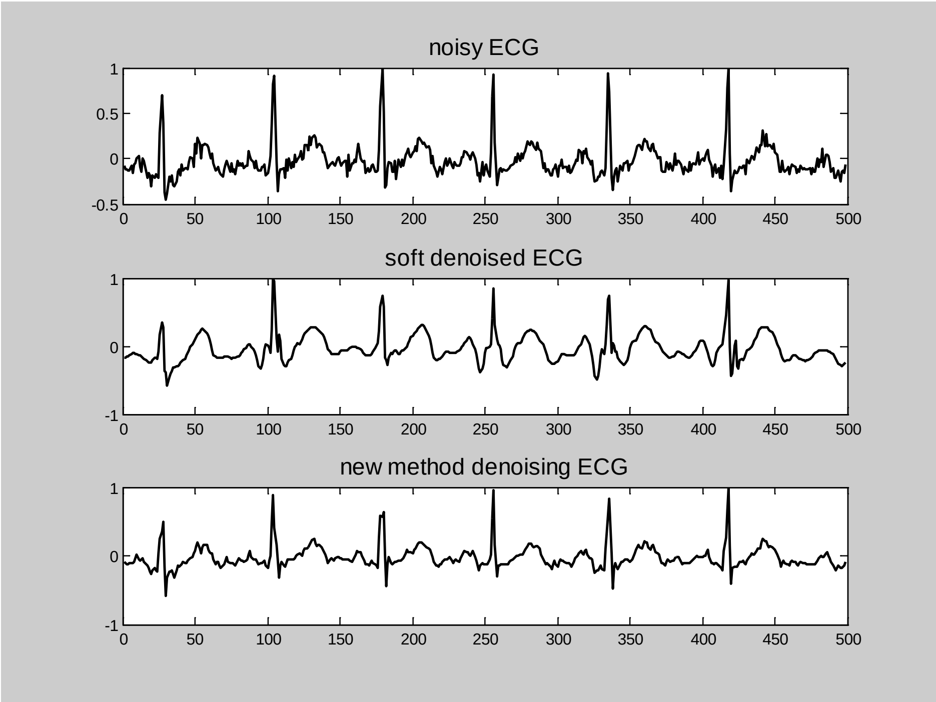

Figure 7: The results obtained using different denoising procedures.

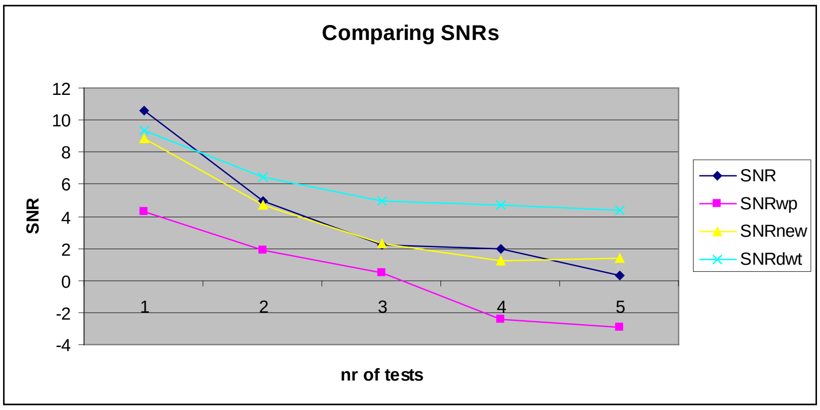

Figure 8: Comparison between signal-to-noise ratios obtained by different denoising methods.

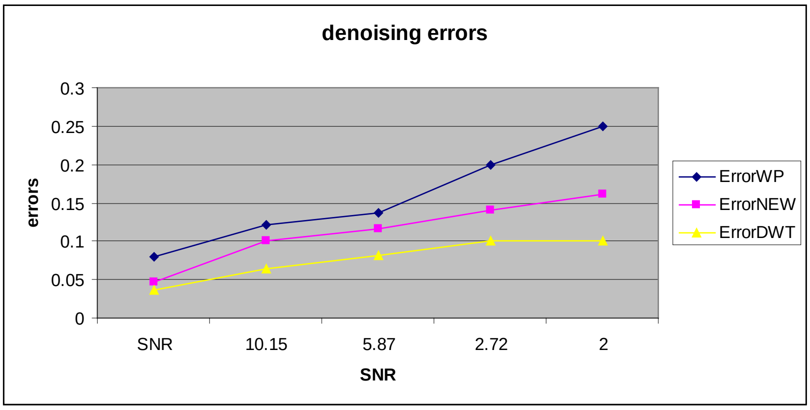

Figure 9: Comparison between different denoising errors.

Figure 7 presents the filtering errors for different methods, the proposed procedure seems to be slightly better than the wavelet packed based denoising method.

5. Conclusions

The main idea was to estimate the correlation between the noise and the signal. The discrete wavelet decomposition algorithm offers a good opportunity to have access to different time-frequency domains in order to perform non-linear filtering. An extra decomposition of the noise was used to reduce this correlation. This method was compared with ordinary wavelet decomposition and wavelet packet decomposition based filtering techniques. Wavelet and wavelet packets based denoising methods gave different performances, due to the different division strategies of the signal decomposition structures.

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