Acta Universitatis Sapientiae
Electrical and Mechanical Engineering, 2 (2010) 166-176

# Kinematic Analysis of a 6 DOF 3-PRRS Parallel Manipulator

Zoltán FORGÓ

Department of Mechanical Engineering, Faculty of Technical and Human Sciences, Sapientia University, Tg. Mureş, e-mail: zforgo@ms.sapientia.ro

Manuscript received October 14, 2010; revised November 08, 2010.

**Abstract:** The number of parallel mechanism applications in the industry is growing and the interest of the academia to find new solutions and applications to implement such mechanisms is present all over the world. In this paper, after a summarised group theory presentation, a symmetrical six degrees of freedom mechanism (3-PRRS) will be defined using this theory. Enumerating some possible kinematic chains for Schoenflies-motion, one solution is kept in order to build up the proposed mechanism. The easy way of mathematical modelling is given by the fact that the mechanism can be considered as an extended well known planar Delta manipulator. The double driven joints in each limb ensure the third translation and other two rotations of the moving platform complementing the planar motion of the Delta manipulator. After the kinematical modelling of the presented mechanism, the actuation of the links is considered. A new parallel driven actuation system is presented in order to fulfill the rotation and translation movements required for the PRRS limb actuation. The aspects of singular configurations, which are similar to the planar Delta mechanism singular configurations with some extensions, are considered also in the presented paper. The paper closes by enumerating some major advantages of the proposed 6 degrees of freedom manipulator.

**Keywords:** parallel mechanism, kinematics, group theory, Lie algebra.

## 1. Introduction

The number of applications in the industry which use parallel mechanisms are growing and the interest of the academia to find new solutions and applications to implement such mechanisms is present all over the world. The lower degree of freedom mechanisms which are suited for some specific tasks are preferred because of the architecture simplicity and therefore the easy mathematical modeling and finally, but not at least for economical reasons.

The 6 degrees of freedom (DOF) parallel mechanism is introduced by Steward and Gough [1] and since then many aspects of the mechanism and its application are revealed. During the last decades more attention has been paid to the study of 6 DOF parallel mechanisms, including synthesis and analysis on kinematics, dynamics, singularities, error and workspace. Some milestones in the analysis of those mechanisms are set by Earl and Rooney using a method for synthesis of new kinematic structures [2], Hunt studied the manipulators on the basis of screw theory [3], Tsai is using systematic methodology in [4] and Hervé discussed the structural synthesis of parallel robots using the mathematical group theory [5]. More recently Shen proposed a systematic type synthesis methodology for 6 DOF kinematic structures enumerating 29 parallel structures [6]. Hereby Shen defines the hybrid single-open chains (HSOC) which are able to generate three translations and three rotation angles. Using those HSOCs four 6 DOF manipulators are presented with symmetrical arrangement of the limbs (see No.3-No.6 architectures, *Table 2*. from [6]). According to Tsai [7], the symmetry implies the use of the same number of actuators on the same positions in each limb. Moreover he says that a parallel manipulator is symmetrical if it satisfies the condition that the number of limbs is equal to the number of degrees of freedom of the moving platform. In the case of double actuated limbs (with two actuated joints) the last presented condition can be omitted. So the HSOCs defined by Shen can be replaced by serial chains which enable three translations and three rotations also.

This paper presents some kinematic structures according to the above mentioned criteria without the aim of full discussion about all the possible structures. The geometrical model of one architecture is presented as well.

## 2. General motion generators

The enumeration of serial topology limbs which enable the spatial motion (three translations and three rotations) is greatly simplified by using the Lie group of rigid body displacement as introduced by Hervé [8]. If each limb of a parallel manipulator generates a subset of possible displacements, which is a Lie subgroup, the intersection set is also a Lie subgroup of the mobile platform. According to this statement if the platform undergoes the spatial, general motion, each limb must ensure the three translations and three rotations. According to *Table 1* $\{D\}$ denotes the general rigid body motions for the 6 DOF mobile platform and $\{L_i\}$ denotes the displacement Lie subgroup of the i$^{th}$ limb. The relation between them is given by:

$$\{L_1\} \cap \{L_2\} \cap \{L_3\} = \{D\}. \quad (1)$$

It is obvious that the only possibility for a true equation (1) is:

$$\{L_1\} = \{L_2\} = \{L_3\} = \{D\}. \quad (2)$$

To obtain simple mechanical structures, better symmetry and good manufacturing for the three limbs the same architecture is considered. For this reason, the analysis of the $\{L_i\}$ displacement Lie subgroup is carried out. The notations for displacement Lie subgroups are recalled in *Table 1* [9].

According to the group theory it can be stated:

$$\begin{aligned}\{L_i\} &= \{D\} = \{T\}\{S(N)\} = \\ &= \{T(\boldsymbol{u})\}\{T(\boldsymbol{v})\}\{T(\boldsymbol{w})\}\{R(N,\boldsymbol{u})\}\{R(N,\boldsymbol{v})\}\{R(N,\boldsymbol{w})\} \quad \forall N.\end{aligned} \quad (3)$$

*Table 1*: List of displacements Lie subgroups [4].

| *Lie subgroup* | *Description of the subgroup* |
|---|---|
| $\{E\}$ | identity |
| $\{T(\boldsymbol{u})\}$ | translations parallel to the ***u*** vector |
| $\{R(N,\boldsymbol{u})\}$ | rotations around the axis determined by N and ***u*** |
| $\{H(N,\boldsymbol{u},p)\}$ | helical motions with axis (N,***u***) and the pitch p |
| $\{T(Pl)\}$ | translations parallel to the *Pl* plane |
| $\{C(N,\boldsymbol{u})\}$ | cylindrical motions along an axis (N,***u***) |
| $\{T\}$ | spatial translations |
| $\{G(\boldsymbol{u})\}$ | planar gliding motions perpendicular to ***u*** |
| $\{S(N)\}$ | spherical motions about point *S* |
| $\{X(\boldsymbol{u})\}$ | Schoenflies motions |
| **{*D*}** | general rigid body motions or displacements |

Based on [9] a planar joint has 5 equivalencies as presented below:

$$\{G(\boldsymbol{u})\} = \{R(A,\boldsymbol{u})\}\{R(B,\boldsymbol{u})\}\{R(C,\boldsymbol{u})\}; \quad (4)$$

$$\{G(\boldsymbol{u})\} = \{R(A,\boldsymbol{u})\}\{T(\boldsymbol{v})\}\{R(C,\boldsymbol{u})\} \quad \boldsymbol{v} \perp \boldsymbol{u}; \quad (5)$$

$$\{G(\boldsymbol{u})\} = \{T(\boldsymbol{v})\}\{R(B,\boldsymbol{u})\}\{R(C,\boldsymbol{u})\} \quad \boldsymbol{v} \perp \boldsymbol{u}; \quad (6)$$

$$\{G(\boldsymbol{u})\} = \{T(\boldsymbol{v})\}\{R(B,\boldsymbol{u})\}\{T(\boldsymbol{w})\} \quad \boldsymbol{v},\boldsymbol{w} \perp \boldsymbol{u}; \quad (7)$$

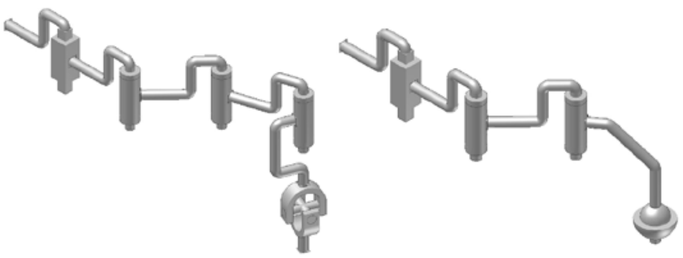
$$\{G(\boldsymbol{u})\} = \{T(\boldsymbol{v})\}\{T(\boldsymbol{w})\}\{R(C,\boldsymbol{u})\} \quad \boldsymbol{v},\boldsymbol{w} \perp \boldsymbol{u}. \quad (8)$$

Considering equations (4) and (8) respectively equality $N \equiv C$ the equation (3) becomes:

$$\{L_i\} = \{T(\boldsymbol{u})\}\{R(A,\boldsymbol{u})\}\{R(C,\boldsymbol{u})\}\{R(B,\boldsymbol{u})\}\{R(B,\boldsymbol{v})\}\{R(B,\boldsymbol{w})\}\ \forall A,B,C; \quad (9)$$

$$\{L_i\} = \{T(\boldsymbol{u})\}\{R(A,\boldsymbol{u})\}\{R(C,\boldsymbol{u})\}\{S(B)\} \quad \forall A,B,C. \quad (10)$$

The above defined $\{L_i\}$ displacement Lie group variants are presented in *Fig. 1*.



a) b)

*Figure 1*: The $\{L_i\}$ displacement Lie group variants incorporating the X-motion generator.

The X-motion (or Shoenflies motion) generator can be easily observed, due to equation (9) and *Fig. 1a*. Considering primitive Schoenflies-motion generators [10] equivalences can be applied. Extending those generator family members with the universal joint as seen in *Fig. 1*, new generators for {D} displacement Lie group can be introduced. However, this enumeration is out of the topic of this paper. Because of the reduced link number and simplicity, in further investigation, the *Fig. 1b* variant is preferred. Using other geometrical constraints the architecture is presented in [11] also. The schematic design of such a limb for a 6 DOF manipulator is presented in *Fig. 2b*. The index *i* is introduced because the same type of limbs are used for moving the manipulator platform.

## 3. Kinematics of 3-PRRS mechanism

The general setup for the parallel mechanism having three translations and three rotations for the end effector (denoted by P) is presented in *Fig. 2c*. For simplicity the mechanism is presented from top view. The geometrical parameters used in the mathematical modelling are enumerated in sketches b) and c) from *Fig. 2*. Further the real number values $x_N$, $y_N$ and $z_N$ are introduced as the coordinates of a point N in the Cartesian space $0x_0y_0z_0$.

Using the equivalency between sketches a) and b) from *Fig. 2* it can be stated:

$$\boldsymbol{C_i B_i} = \boldsymbol{C_i D_i} + \boldsymbol{D_i B_i} = C_i D_{ix} \cdot \boldsymbol{i} + C_i D_{iy} \cdot \boldsymbol{j} + D_i B_i \cdot \boldsymbol{k} \,, \tag{11}$$

$$\boldsymbol{C_i B_i} = C_i B_{ix} \cdot \boldsymbol{i} + C_i B_{iy} \cdot \boldsymbol{j} + D_i B_i \cdot \boldsymbol{k} \,, \tag{12}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors of the $x_0$, $y_0$ and $z_0$ Cartesian axes. The setup of the mechanism (based on the projection of the manipulator on the $0x_0y_0$ plan – top view from *Fig. 2*) suggests a planar Delta manipulator.

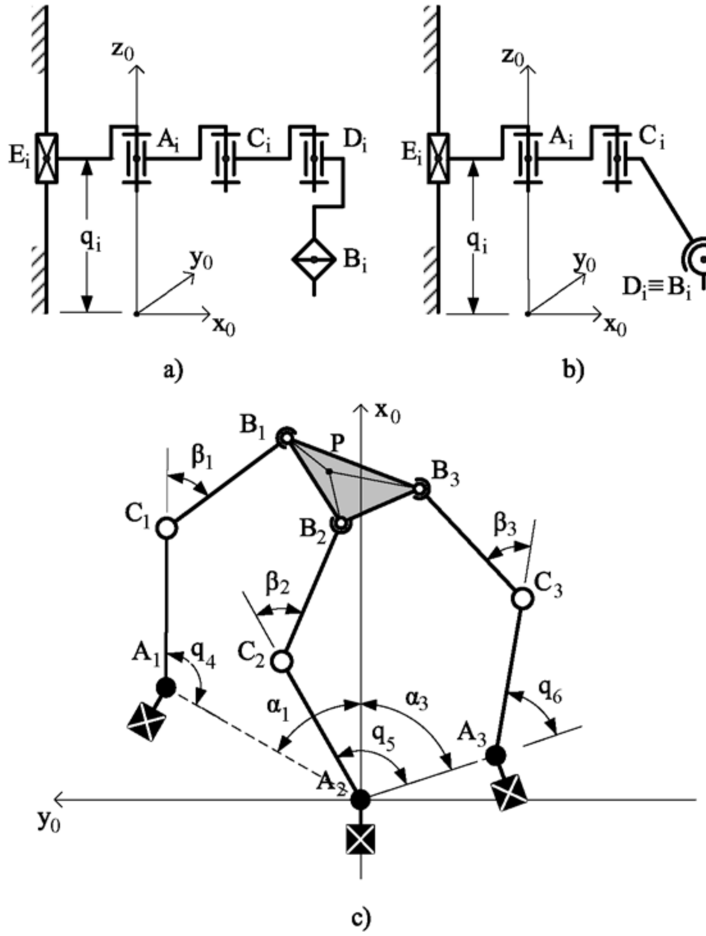



*Figure 2*: Schematic design of i$^{th}$ limb of the 6 DOF manipulator (a, b), and the top view of the proposed mechanism (c). The shaded couplings are the active joints (one prismatic and one rotation for each limb), and the white ones are passive bonds.

For this reason the mathematical modelling of the proposed mechanism is made easily and it is like a well known planar Delta manipulator modelling [7] with some completions. These completions must be made due to the fact, that it is possible to rotate the platform around the $x_0$ and $y_0$ axes too, and so the projections of the $B_iB_{i+1}$ platform length are variable. Through these paragraphs the inverse and direct kinematics of the proposed mechanism is defined, and issues about singular configurations are presented as well.

At the beginning the closure equation is considered for the three limbs:

$$\boldsymbol{OA_i} + \boldsymbol{A_iC_i} + \boldsymbol{C_iB_i} = \boldsymbol{OP} + \boldsymbol{PB_i} \text{ where } i = 1,2,3 . \tag{13}$$

In case of inverse kinematic modelling the right side of equation (13) is given through the coordinates of the characteristic point (denoted by $P$) and through the three rotation angles around the axes of the fixed $0x_0y_0z_0$ system: $\boldsymbol{X} = [x_P \ y_P \ z_P \ \alpha \ \beta \ \gamma]^T$. The task is to determine the robot parameters $\boldsymbol{q} = [q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6]^T$ from the left side of the equation. Assuming that vector $\boldsymbol{a}$ has the components $\boldsymbol{a_{xy}}$ parallel to the $0x_0y_0$ plan and $\boldsymbol{a_z}$ parallel to the $z_0$ axis, equation (13) becomes:

$$\begin{cases} \boldsymbol{OA_{ixy}} + \boldsymbol{A_iC_{ixy}} + \boldsymbol{C_iB_{ixy}} = \boldsymbol{OP_{xy}} + \boldsymbol{PB_{ixy}} \\ \boldsymbol{OA_{iz}} + \boldsymbol{A_iC_{iz}} + \boldsymbol{C_iB_{iz}} = \boldsymbol{OP_z} + \boldsymbol{PB_{iz}} \end{cases} \text{ where } i = 1,2,3 . \tag{14}$$

In order to determine the $q_i$ translational parameters ($i$=1,2,3), introduced in *Fig. 2b*, the second equation from (14) is considered:

$$q_i \cdot \boldsymbol{k} + \boldsymbol{C_iB_{iz}} = z_P \cdot \boldsymbol{k} + \boldsymbol{PB_{iz}} \text{, respectively in scalar form} \tag{15}$$

$$q_i = z_P + PB_{iz} + C_iB_{iz} \tag{16}$$

Hence $C_iB_{iz}$ is a constant geometrical parameter of the manipulator, the first two terms from the right side of equation (16) contain the general parameters because $PB_{iz} = PB_{iz}(\alpha,\beta,\gamma)$.

To obtain the $q_{i+3}$ rotation joints parameters ($i$=1,2,3) the first equation from (14) is recalled and presented in scalar form:

$$\begin{cases} OA_i\cos\alpha_i + A_iC_i\cos(q_{i+3}+\alpha_i-\pi) + C_iB_i\cos(q_{i+3}+\alpha_i-\pi-\beta_i) = x_P + PB_{ix} \\ OA_i\sin\alpha_i + A_iC_i\sin(q_{i+3}+\alpha_i-\pi) + C_iB_i\sin(q_{i+3}+\alpha_i-\pi-\beta_i) = y_P + PB_{iy} \end{cases}, \tag{17}$$

where $PB_{ix} = PB_{ix}(\alpha,\beta,\gamma)$ and $PB_{iy} = PB_{iy}(\alpha,\beta,\gamma)$ respectively $i$=1,2,3. To eliminate the $\beta_i$ parameter belonging to a passive joint, the equations are rearranged, and summing the square of the two equations in (17) yields:

$$e_{1i} \cdot \sin(q_{i+3}+\alpha_i-\pi) + e_{2i} \cdot \cos(q_{i+3}+\alpha_i-\pi) + e_{3i} = 0 , \tag{18}$$

where

$$\begin{cases} e_{1i} = -2 \cdot A_i C_i \cdot \left( y_P + PB_{iy} - OA_i \sin\alpha_i \right); \\ e_{2i} = -2 \cdot A_i C_i \cdot \left( x_P + PB_{ix} - OA_i \cos\alpha_i \right); \\ e_{3i} = \left( x_P + PB_{ix} - OA_i \cos\alpha_i \right)^2 + \left( y_P + PB_{iy} - OA_i \sin\alpha_i \right)^2 + A_i C_i^{\,2} - C_i B_i^{\,2}. \end{cases} \tag{19}$$

Solving equation (18) by using the substitutions:

$$\begin{cases} \sin\left(q_{i+3} + \alpha_i - \pi\right) = \dfrac{2t_i}{1+t_i^2} \\ \cos\left(q_{i+3} + \alpha_i - \pi\right) = \dfrac{1-t_i^2}{1+t_i^2} \end{cases} \text{ where } t_i = \tan\frac{q_{i+3} + \alpha_i - \pi}{2}, \tag{20}$$

the $q_{i+3}$ parameters ($i$=1,2,3) are given by:

$$q_{i+3} = \pi - \alpha_i + 2\tan^{-1}\frac{-e_{1i} \pm \sqrt{e_{1i}^2 + e_{2i}^2 - e_{3i}^2}}{e_{1i} - e_{2i}}. \tag{21}$$

Equations (16) and (21) define the robot parameters in case of inverse kinematics. To obtain the general coordinates of the mechanism it is necessary to calculate the position of the $B_i(x_{Bi},y_{Bi},z_{Bi})$ joints ($i$=1,2,3) in Cartesian space and knowing the geometrical dimensions of the mobile platform the $\boldsymbol{X} = [x_P \;\; y_P \;\; z_P \;\; \alpha \;\; \beta \;\; \gamma]^T$ vector is obvious. The $x_{Bi}$, $y_{Bi}$, and $z_{Bi}$ values are defined through the following nine equations:

$$\begin{cases} \left(x_{Ci} - x_{Bi}\right)^2 + \left(y_{Ci} - y_{Bi}\right)^2 = C_i B_i^2 \\ \left(x_{Bi} - x_{Bj}\right)^2 + \left(y_{Bi} - y_{Bj}\right)^2 = d^2 - \left(z_{Bi} - z_{Bj}\right)^2 \\ z_{Bi} = z_{Ci} - D_i B_i \end{cases} \text{ for } \begin{array}{l} i = 1,2,3 \\ j = \begin{cases} i+1, if\ i = 1,2 \\ 1\ , if\ i = 3 \end{cases} \end{array} \tag{22}$$

where $x_{Ci} = x_{Ci}(q_{i+3}), y_{Ci} = y_{Ci}(q_{i+3}), z_{Ci} = z_{Ci}(q_i)$ respectively $C_i B_i$ and $D_i B_i$ are constant geometrical dimensions using $i$ = 1,2,3. It is important to mention, that in present case the forward kinematics deals with only 8 solutions (as by the planar Delta robot).

To complete the kinematic calculations the relation between the actuated joints and the platform velocities is needed and obtained through:

$$J_x \cdot \dot{\boldsymbol{X}} = J_q \cdot \dot{\boldsymbol{q}}, \tag{23}$$

where the matrices:

$$J_x = \begin{bmatrix} b_{1x} & b_{1y} & b_{1z} & e_{1y}b_{1z}-e_{1z}b_{1y} & e_{1z}b_{1x}-e_{1x}b_{1z} & e_{1x}b_{1y}-e_{1y}b_{1x} \\ b_{2x} & b_{2y} & b_{2z} & e_{2y}b_{2z}-e_{2z}b_{2y} & e_{2z}b_{2x}-e_{2x}b_{2z} & e_{2x}b_{2y}-e_{2y}b_{2x} \\ b_{3x} & b_{3y} & b_{3z} & e_{3y}b_{3z}-e_{3z}b_{3y} & e_{3z}b_{3x}-e_{3x}b_{3z} & e_{3x}b_{3y}-e_{3y}b_{3x} \end{bmatrix}, \quad (24)$$

$$J_q = \begin{bmatrix} b_{1z} & 0 & 0 & a_{1x}b_{1y}-a_{1y}b_{1x} & 0 & 0 \\ 0 & b_{2z} & 0 & 0 & a_{2x}b_{2y}-a_{2y}b_{2x} & 0 \\ 0 & 0 & b_{3z} & 0 & 0 & a_{3x}b_{3y}-a_{3y}b_{3x} \end{bmatrix}, \quad (25)$$

can be written using the notations $\boldsymbol{a}=\boldsymbol{A_i C_i}$, $\boldsymbol{b}=\boldsymbol{C_i B_i}$ and $\boldsymbol{e}=\boldsymbol{PB_i}$. Equation (23) can be considered for calculation of direct and inverse kinematics.

## 4. Parallel drive actuation of a manipulator limb

To assure the parallel mechanism concept for the 3-<u>PR</u>RS manipulator the parallel drive of the three limbs must be realized. Therefore a toothed belt drive H-shaped system can be applied as it can be seen in *Fig. 3*. At the bottom of the mechanism the actuated pulleys (gray color fill) induce the motion in the mechanism by the $q_i^M$ and $q_{i+3}^M$ driving parameters. The values $q_i$ and $q_{i+3}$ are set as the output parameters.

Using the parallel drive system two rotation inputs are transformed into translation and rotation output. The relation between them is given by the following equation:

$$\begin{bmatrix} q_i \\ q_{i+3} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & -\frac{r}{2} \\ -\frac{r}{2R} & -\frac{r}{2R} \end{bmatrix} \cdot \begin{bmatrix} q_i^M \\ q_{i+3}^M \end{bmatrix}. \quad (26)$$

The inverse geometry calculus can be performed using the following equation:

$$\begin{bmatrix} q_i^M \\ q_{i+3}^M \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & -\frac{R}{r} \\ -\frac{1}{r} & -\frac{R}{r} \end{bmatrix} \cdot \begin{bmatrix} q_i \\ q_{i+3} \end{bmatrix}. \quad (27)$$

Using the above formulation and considering equations (16) and (21) the inverse geometry is obtained in the following form:

$$\overline{q}^M = \begin{bmatrix} q_1^M \\ q_2^M \\ q_3^M \\ q_4^M \\ q_5^M \\ q_6^M \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & 0 & 0 & -\frac{R}{r} & 0 & 0 \\ 0 & \frac{1}{r} & 0 & 0 & -\frac{R}{r} & 0 \\ 0 & 0 & \frac{1}{r} & 0 & 0 & -\frac{R}{r} \\ -\frac{1}{r} & 0 & 0 & -\frac{R}{r} & 0 & 0 \\ 0 & -\frac{1}{r} & 0 & 0 & -\frac{R}{r} & 0 \\ 0 & 0 & -\frac{1}{r} & 0 & 0 & -\frac{R}{r} \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = A \cdot \overline{q} \, . \quad (28)$$

Due to the characteristic setup of the driving mechanism the equations for the kinematics are obtained in similar way:

$$\dot{\overline{q}}^M = A \cdot \dot{\overline{q}} \text{ and } \dot{\overline{q}} = A^{-1} \cdot \dot{\overline{q}}^M \, . \quad (29)$$

In accordance with the formulated equations the dynamics of the manipulator can be calculated easily, and will be presented in a further paper.

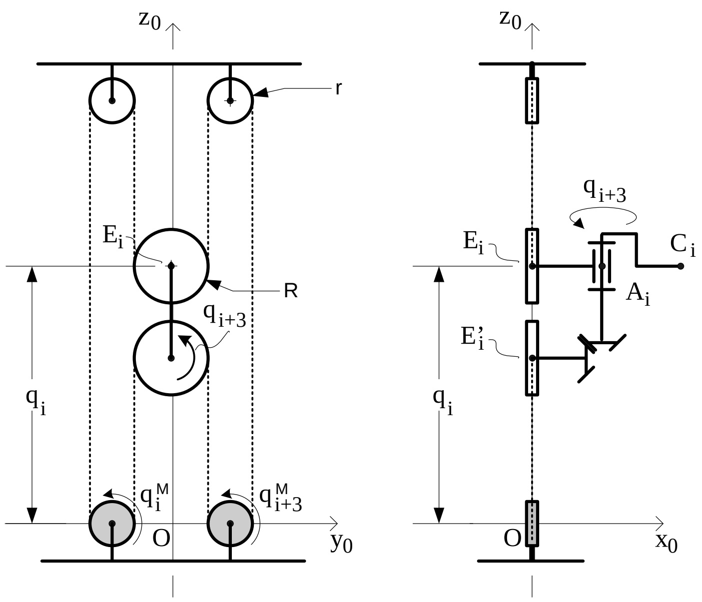



*Figure 3:* Schematic design of the belt mechanism for one, double drive link.

## 5. Singular configurations

The singularity analysis of this mechanism can be done based on the matrices from (24) and (25). Inverse kinematic singularities occure in case of $a_{ix}b_{iy} - a_{iy}b_{ix} = 0$ ($i$=1,2,3) which defines the workspace boundaries. An other possibility is $b_{iz} = 0$ ($i$=1,2,3) but it can be avoided through geometrical design, because it is a constant value. Direct kinematic singularities occure when at the same time it can be stated that $b_{ix} = 0$ or $b_{iy} = 0$ ($i$=1,2,3), which means that the $C_iB_i$ links are parallel. The same type of singularities can be found for coexistence of $e_{ix}b_{iy} - e_{iy}b_{ix} = 0$ ($i$=1,2,3) in case of coliniar $\boldsymbol{C_i B_i}$ and $\boldsymbol{PB_i}$ vectors. Both direct kinematic singularity cases can be avoided by careful geometrical design. The implemented parallel drive mechanisms have no singular configurations, and this kind of calculations can be omitted.

## 6. Conclusions

This paper deals with a 6 degrees of freedom manipulator architecture using the group theory. The mobile platform is connected to the base through three PRRS limbs, each being double actuated on the first and second joint levels. The inverse geometrical calculations are performed through equations (16) and (21), hence the direct modelling is presented through the equation system (22). The relation between the robot and general velocities is stated by the equation (23). Some aspects about the singular configurations are introduced in the paper based on the equation mentioned before. As it can be seen in the figures presented in this paper the architecture is the extension of the well known planar Delta robot to a 6 DOF mechanism. The mathematical model of the spatial manipulator reflects this fact very well. The simple setup of the presented mechanism assures a good manufacturability and needs a relatively easy control algorithm considering some other 6 DOF manipulators.

## References

[1] Stewart, D. A., "Platform with six degrees of freedom", in *Proceedings on Institution of Mechanical Engineering*, 1965, vol. 180, pp. 371-386.

[2] Dasguta, B. and Mruthyunjaya, T. S., "The Stewart platform manipulator: a review" *Mechanism and Machine Theory*, vol. 35, pp. 15-40, 2000.

[3] Hunt, K. H., "Structural kinematics of in-parallel-actuated robot arms", *ASME Journal of Mechanical Design*, vol. 105, pp. 705-712, 1983.

[4] Tsai, L. W., and Joshi, S., "Kinematics and optimization of a Spatial 3-UPU Parallel manipulator", *ASME Journal of Mechanical Design*, vol. 122, pp. 439-446, 2000.

[5] Hervé, J. M., "Design of parallel manipulators via the displacement group", in *Proceedings of the 9th World Congress on Theory of Machine and Mechanisms*, Milano, 1985, pp. 2079-2082.
[6] Shen, H., Yang., T., and Ma, L., "Synthesis and structure analysis of kinematic structures of 6-DOF parallel robotic mechanisms", *Mechanism and Machine Theory*, vol. 40, pp. 1164-1180, 2005.
[7] Tsai, L. W., "Robot Analysis – The Mechanics of Serial and Parallel Manipulators", John Wiley & Sons, 1999.
[8] Hervé, J. M., "The Lie group of rigid body displacements, a fundamental tool for mechanism design", *Mechanism and Machine Theory*, vol. 34, pp. 719-730, 1999.
[9] Hervé, J. M., "The planar-spherical kinematic bond: Implementation in parallel mechanisms", http://www.parallemic.org/Reviews/Review013p.html.
[10] Lee, C. C. and Hervé, J. M., "Type synthesis of primitive Schoenflies-motion generators", *Mechanism and Machine Theory*, vol. 44, pp. 1980-1997, 2009.
[11] Olea, G., Plitea, N., and Takamusa, K., "Kinematical analysis and simulation of a new parallel mechanism for robotics application", in *ARK Piran, Piran*, 25-29 June 2000, pp. 403-410.