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Incremental Encoder Based Position and Speed Identification: Modeling and Simulation

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Abstract: Electrical drives frequently use incremental encoders as position sensor. The paper deals with the modeling and simulation of an incremental encoder and associated units for processing the information provided by the encoder. A mathematical model of the incremental encoder is presented. Based on encoder signals the direction of the rotation, the position and the speed are identified. The described procedure for determination of the direction of the rotation is able to identify the direction changing in all cases during a rotation equal to the minimal detectable rotation-angle-increment. The computing of the position is based on the algebraic summing of the number of the generated encoder signals. For the speed determination different methods are modeled and simulated: for high speed region the frequency measurement is used and for low speed domain the period measurement is appropriate. In case of a large speed variation the minimal-error-based switching between the two methods is suitable. Matlab-Simulink® simulation structures were realized for the encoder signals based on the identification of the direction of the rotation, for the position and speed computation. Experimental results performed on a DSP-based set-up (under development) are given. The presented simulation subsystems of the encoder, position and speed computation may be integrated in any Matlab-Simulink® structure.

Keywords: Angle transducer, position sensor, incremental encoder modeling, incremental encoder simulation, angular speed identification, electrical drive.

1. Introduction

The incremental encoder is a device which provides electrical pulses if its shaft is rotating [1], [2], [4]. The number of the generated pulses is proportional to the angular position of the shaft. The incremental encoder is one of the most frequently used position transducers. The principle of an optical incremental encoder is presented in *Fig. 1*. Together with the shaft there is rotating a transparent (usually glass) rotor disc with a circular graduation-track realized as a periodic sequence of transparent and non-transparent radial zones which modulates the light beams emitted by a light source placed on one side of the disc on the fix part (stator) of the encoder. On the opposite side the modulated light beams are sensed by two groups of optical sensors and processed by electronic circuits. Each of the two outputs of the encoder (noted *A* and *B*) will generate one pulse when the shaft rotates an angle equal to the angular step of graduation θ_p, i.e. the angle according to one successive transparent and non-transparent zone. The number of pulses (counted usually by external electronic counters) is proportional to the angular position of the shaft. Due to the fact that the light beams are placed shifted to each other with an angle equal to the quarter of angular step of graduation $\theta_p/4$, the pulses of the two outputs will be also shifted, making possible the determination of the rotation sense. A third light beam is modulated by another track with a single graduation. The output signal (named *Z*) associated to this third beam provides a single pulse in the course of a complete (360°) rotation. The shaft position corresponding to this pulse may be considered as reference position. *Fig. 2* shows the output pulses of the encoder.

Usually for counter-clockwise (CCW) direction θ is considered as positive, and for clockwise (CW) direction it is considered negative.

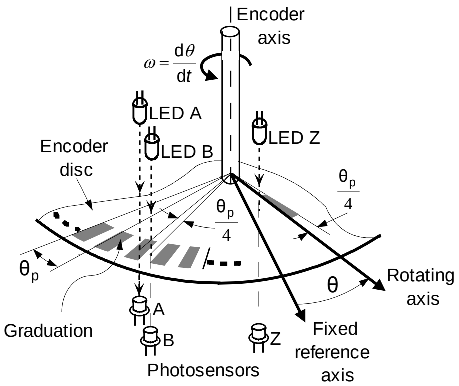


Figure 1: Construction principle of the incremental encoder: the gray surfaces are optically transparent.

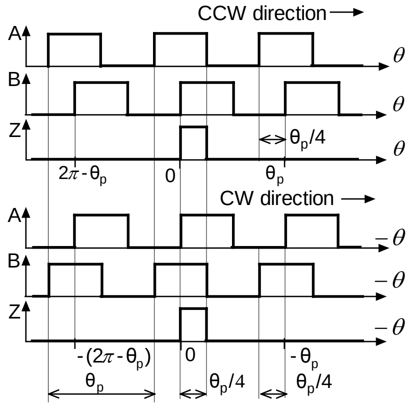


Figure 2: Diagram of the output signals for counter-clockwise (CCW) and clockwise (CW) rotation.

2. Incremental encoder modeling

The input signal of the incremental encoder is the angular position θ of its shaft with respect to the fixed reference axis. The output signals are the two pulses shifted by a quarter angular step $A(\theta)$ and $B(\theta)$, respectively the marker signal $Z(\theta)$. If θ_p is the angular step of the encoder, the outputs may be described by the following equations [2]:

$$A(\theta)=\begin{cases}1 & \text{if} \quad 0 \le (\theta \bmod \theta_p) \le \theta_p/2;\\ 0 & \text{if} \quad \theta_p/2 < (\theta \bmod \theta_p) \le \theta_p;\end{cases}$$
$$B(\theta)=\begin{cases}1 & \text{if} \quad 0 \le ((\theta - \theta_p/4) \bmod \theta_p) \le \theta_p/2;\\ 0 & \text{if} \quad \theta_p/2 < ((\theta - \theta_p/4) \bmod \theta_p) \le \theta_p;\end{cases} \tag{1}$$
$$Z(\theta)=\begin{cases}1 & \text{if} \quad \theta \bmod (2\pi) = 0;\\ 0 & \text{if} \quad \theta \bmod (2\pi) \ne 0.\end{cases}$$

During a rotation angle of the shaft, equal to the angular step of graduation θ_p, there are four switching events in the output pulses; therefore the minimal rotation-angle-increment detectable by the encoder is $\theta_p/4$ [3]. The number of pulses, generated by the encoder in the course of a rotation, is equal with the number of angular steps of the graduation on the circular track on the rotor.

$$N_r = \frac{2\pi}{\theta_p} \tag{2}$$

Based on (1) a Matlab/Simulink® a simulation structure shown in *Fig. 3* was built. The outputs A, B and Z are computed by Simulink® function blocks. θp is defined by a constant block. The structure is saved as a subsystem. The simulation structure of the incremental encoder may be integrated in any other Simulink® structure.

3. Encoder based position identification

In an incremental-encoder-based system the angular position θ is measured with respect to a fixed reference axis (θ=0 rad.) and it is obtained by algebraic counting of the number of the generated encoder pulses according to the CCW (ΣN_i pulses) and CW direction (ΣN_j pulses) and multiplying it with the angular step θ_p of the encoder [2]. Mathematically:

$$\theta = \theta_p \left(\sum_i^{CCW} N_i - \sum_j^{CW} N_j \right) = \theta_p N \ . \qquad (3)$$

In order to compute the algebraic number of pulses it is necessary to know the direction of the rotation.

A. Identification of the rotation direction

The two $\theta_p/4$ shifted output signals of the encoder contain implicitly also the direction information which may be obtained in different ways.

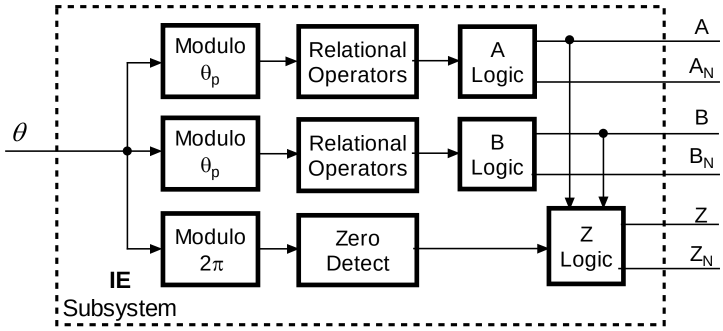


Figure 3: Simulation structure of the incremental encoder.
(Note: The subscript "N" denotes the negated logical variable.)

Taking into account the all four possible combinations of *A* and *B* signals for the reversals, it is possible to detect the direction changing in all cases during a rotation of the minimal detectable rotation-angle-increment $\theta_p/4$. *Table 1* shows the all combinations of signals which detect the reversal of the rotation sense.

Table 1. Combination of signals which detect the reversal of rotation.

From CCW to CW Q=1 to Q=0 (Triggered by R)		From CW to CCW Q=1 to Q=0 (Triggered by S)	
Occurs if		Occurs if	
A	*B*	*A*	*B*
0	0→1	0→1	0
0→1	1	0	1→0
1→0	0	1	0→1
1	1→0	1→0	1

Note: The 0→1 denote the raising-edge and the 1→0 the falling-edge of associated logic variable.

A trivial solution of the problem may be sampling at every rising edge of the *B* output pulses of the *A* output logic value. The resulted logic value will be 1 for counterclockwise (CCW) direction of rotation, and 0 for the clockwise (CW) direction. The method detects the direction changing only after a time interval according to a rotation of 3 $\theta_p/4 - 5\ \theta_p/4$.

B. The position-identification structure

Based on (3) and the above presented direction identification method, a Simulink® subsystem was built for position computation. Its structure is presented in *Fig. 4*.

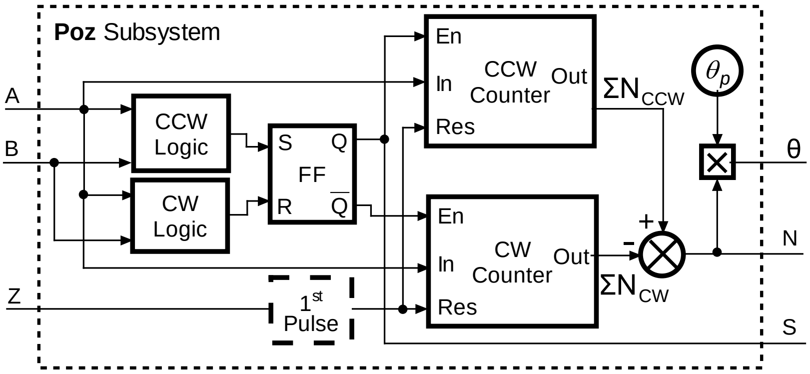


Figure 4: The simulation structure of the position computing subsystem.

The left side of the structure identifies the direction of the rotation. The direction signal S enables the appropriate (CCW or CW) counter. The structure has to be provided with the "1st Pulse" block (broken line in *Fig. 4*) in order to extend the position measurement to more rotations.

4. Encoder based speed identification

The signals generated by incremental encoder provide also information regarding the speed of rotation. A few basic methods are known which are discussed below.

A. Speed identification based on frequency measurement

The frequency of the encoder pulses is proportional to the angular speed. The number of pulses ΔN is counted during a known fix sampling period Ts (see *Fig. 5*) [5]. The angular speed is determined by the expression:

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \cong \frac{\mathrm{d}\theta}{T_s} \cong \frac{2\pi\Delta N}{N_r T_s} = \frac{\theta_p \Delta N}{T_s} \tag{4}$$

Due to the lack of synchronization between the sampling period and encoder pulses a quantization error occurs. The relative error of the procedure is given by

$$\varepsilon_f = \frac{1}{\omega} \frac{2\pi}{N_r T_s} \tag{5}$$

and depends on the reciprocal of the speed, the measuring interval and the resolution of the encoder [5].

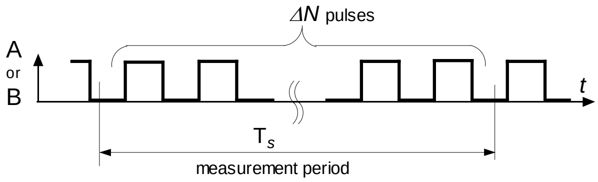


Figure 5: The principle of the speed identification based on the frequency measurement.

The speed calculation structure based on frequency measurement is presented in *Fig. 6*. In order to enhance the precision, the “Logic x4” block multiplies by 4 the frequency of the encoder signals. Two, alternatively resetted and enabled counters count the number of pulses. The content of the just disabled counter is used for speed computation.

The speed identification based on frequency measurement produces relatively small errors at high speed because the number of pulses from the encoder in the measurement-time interval is high.

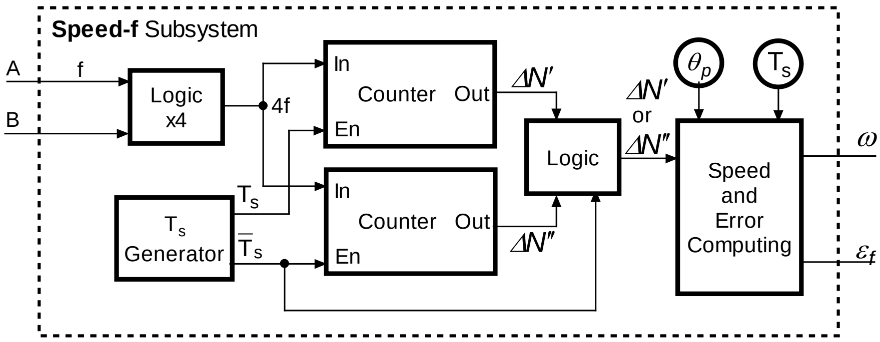


Figure 6: The simulation structure of the speed computing subsystem based on frequency measurement.

B. Speed identification based on period measurement

The speed identification based on frequency measurement at low speed is no longer an option. A better solution is the speed identification based on period measurement. The method consists of counting the pulses of a high frequency clock signal (having period Thf) during an encoder period, as is shown in *Fig. 7* [5].

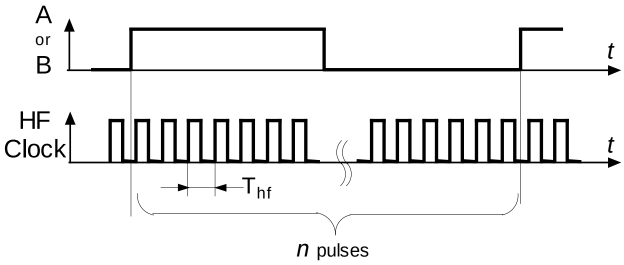


Figure 7: The principle of the speed identification based on the period measurement.

In this case the expression of the angular speed is

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \cong \frac{\mathrm{d}\theta}{nT_{hf}} \cong \frac{2\pi}{N_r n T_{hf}} \tag{6}$$

where n represents the counted number of the high frequency pulses. The relative error is increasing with the rotation frequency and is given by [5]

$$\varepsilon_p = \frac{N_r \omega T_{hf}}{2\pi} \tag{7}$$

The speed calculation structure based on period measurement is presented in *Fig. 8*.

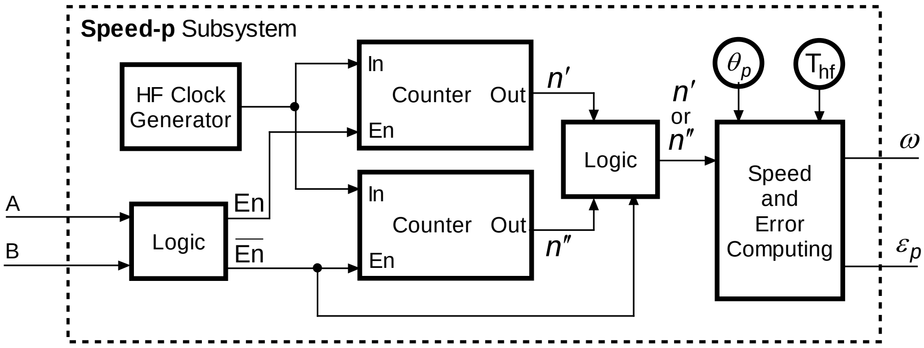


Figure 8: The simulation structure of the speed computing subsystem based on the period measurement.

The structure resembles the previous one; the main difference is that the high frequency pulses are counted during an encoder signal period.

C. Combined method for speed identification

In order to minimize the speed identification error it is suitable to use in low speed region the period measurement method $u(t)$ and as the speed is increasing to switch to the frequency measurement one. The moment of switching is given by the equality of the errors ($\varepsilon_p = \varepsilon_f$), that happens when the speed riches the value

$$\omega_s = \frac{2\pi}{N_r} \cdot \frac{1}{\sqrt{T_s \cdot T_{hf}}} \tag{8}$$

The subsystem presented in *Fig. 9* computes the speed based on above described method.

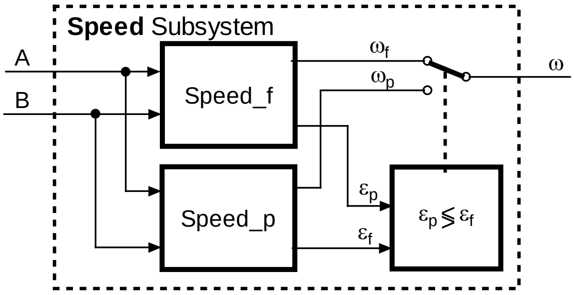


Figure 9: The simulation structure of the combined speed computing subsystem.

The compensation of the errors caused by the sampling times may enhance the precision of the measurement [6].

5. Simulation results

The structure of the interconnected functional units for simulation of position computing is shown in *Fig. 10*. The reference angular position θ_{ref} (the input signal of the encoder block "IE") is generated by a user programmable "Function generator" block. The encoder generates the A, B and Z signals. Based on these, the block "Poz" computes the position θ and the block "Speed" provide the computed angular speed. In order to test the structure, the function generator was programmed in order to start the simulation generating a positive ramp-reference angle, which is the input signal for the "IE" encoder block. At 0.2 s the ramp is switched to negative (equivalent to a reversal from CCW to CW), decreasing in time until 0.8 s, when it is again switched to positive.

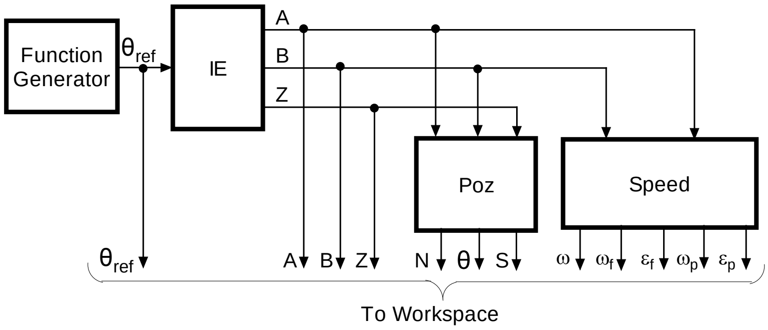
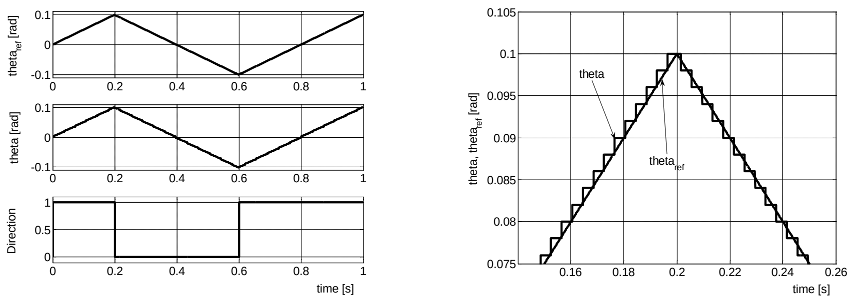


Figure 10: The structure of the interconnected functional units for simulation of the position and speed computation:
IE – incremental encoder, Poz – position computing block,
Speed – speed computing block.

The time profile of the generated reference angle is presented in *Fig. 11 a)* (top trace). The block "Poz", using the encoder output signals, determines the direction of the rotation (in *Fig. 11 a*) bottom trace) and computes the position (shown in *Fig. 11 a)* middle trace). The computed position follows very well the reference one. *Fig. 11 b)* presents an enlarged detail of superposed reference and computed angle before and after the reversal at 0.2 s. The incremental character of the computed position is evident.



a) b)

Figure 11: The simulation results representing the reference angle and computed angle during reversal.
a) From top to bottom: reference angle θ_{ref}, computed angle θ, direction signal S.
b) Detail of the reference angle θ_{ref} and computed angle θ versus time.

There was analyzed the reversal process. The simulated results are presented in *Fig. 12. a)–d)*. The parameters of the function generator were selected in such a manner, that all possible combinations of signals A and B at reversal (presented in *Table 1*) were captured. In all cases the sensing of the reversal is done in a quarter of angular step, as is shown in *Fig. 12.*

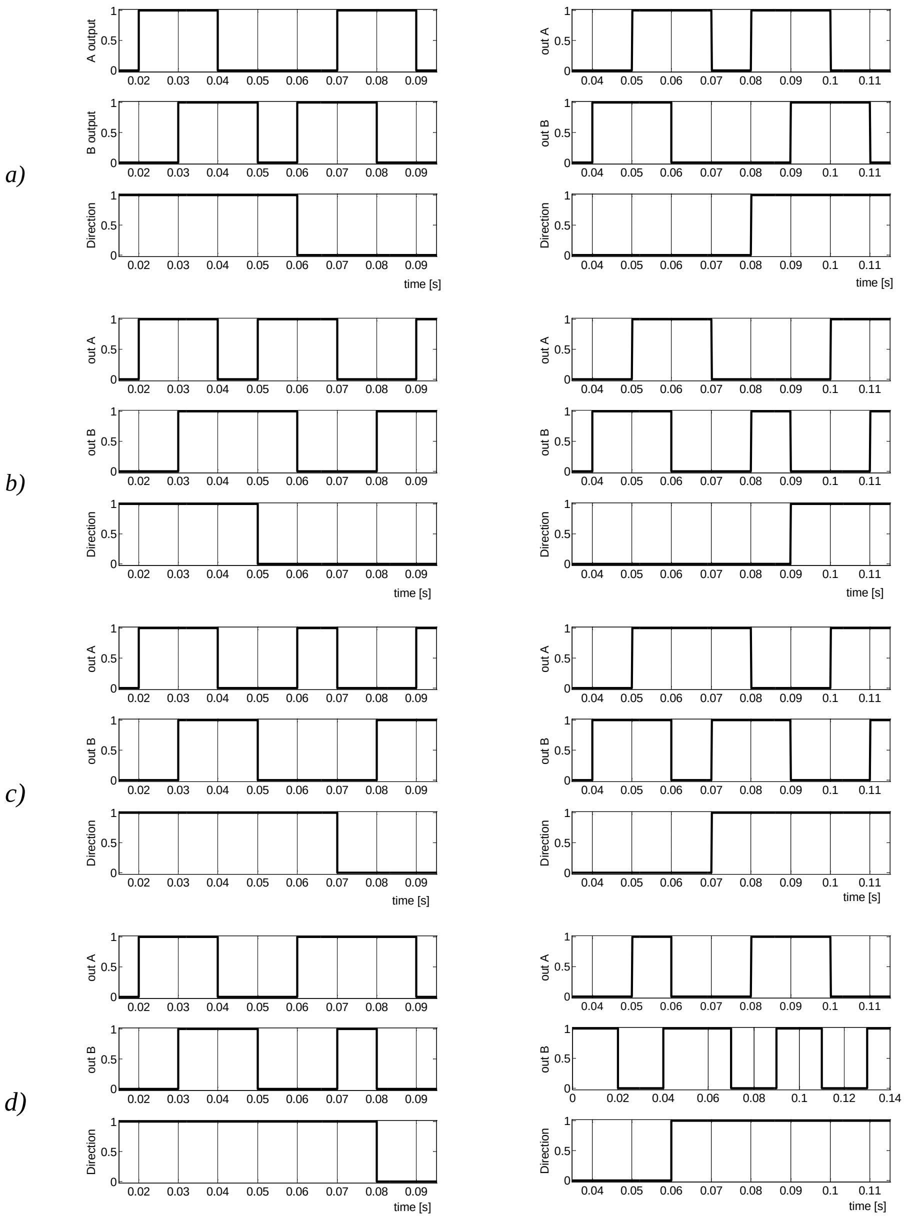
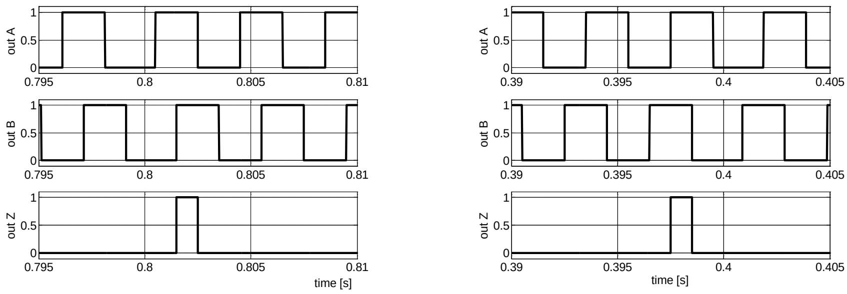


Figure 12: The simulation results showing all combinations of the reversal process. Left column: Reversal from CCW to CW, Right column: Reversal from CW to CCW, top trace: output A, middle trace: output B, bottom trace: direction signal. Reversal occurs at: a) A=0, B=0; b) A=0, B=1; c) A=1, B=0; d) A=1, B=1.

Fig. 13 presents the A, B and Z signals at crossing the reference position (θ = 0) in CW and CCW direction.



a) b)

Figure 13: The A, B and Z signals of the encoder at crossing the reference position: a) in CCW direction, b) in CW direction.

In order to test the speed identification, the function generator was programmed for a linearly increasing and decreasing speed profile. *Fig. 14* shows the theoretical speed profile and the computed speed. In *Fig. 15* is presented the variation of the errors ε_f and ε_p. The switching between the two methods occurs at moment 0.2 s and 0.8 s, respectively.

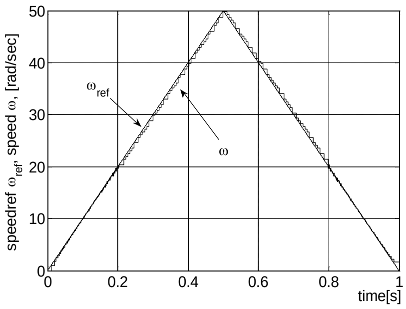


Figure 14: The simulation results showing reference and calculated speed.

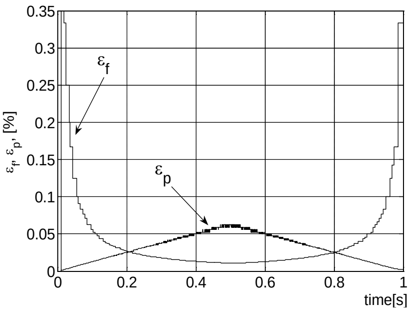


Figure 15: Variation of the error versus time of the two speed calculation methods.

The structure presented in *Fig. 10* may be integrated in the simulation structures of electrical drives [2], [5]. In this case the input signal of the encoder – i.e. the angular position – will be provided by the mathematical model of the electrical machine. The computed position and speed is used as position feed-back signal by the control system of the drive.

The conditions used in simulations are: N=500, T_{hf} = 4 μs, T_s = 6 ms, the simulation step was taken 1 μs.

6. Experimental results

In order to investigate different position and speed determination algorithms an experimental set-up is under construction (see *Fig. 16*). The incremental encoder (type 1XP8001-1) is mounted on the shaft of an induction motor driven by a static frequency converter. (Micromaster, *Siemens*). The encoder signals are processed by an experimental board built around a DSP based development board from Spectrum Digital.

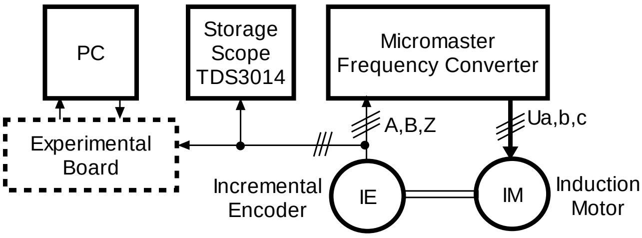
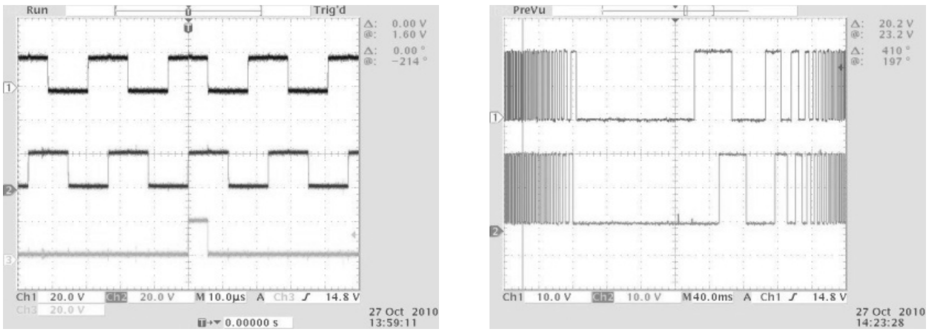


Figure 16: Block scheme of the experimental rig.

Fig. 17 a) shows the captured encoder signals for CCW rotation, and *Fig. 17 b)* represents the A and B signals during the CW to CCW reversal process. The reversal occurs for A=0 B=0.



a) b)

Figure 17: Captured encoder output signals
a) for CCW direction versus time;
Top: Signal A, Middle: Signal B, Bottom: Marker signal Z;
b) for direction reversal from CW to CCW direction versus time;
Top: Signal A, Bottom: Signal B.

A comparison of the above figures to *Fig. 13 a)* and *Fig. 12 a)* shows that the captures are very closed to the simulated results.

7. Conclusion

The information provided by the incremental encoders is inherently digital.

The angular position of the encoder shaft is obtained by algebraic summing of the number of pulses provided by the encoder according to CCW and CW rotation.

The direction of the rotation may be determined by a digital decoding scheme using the two quadrature signals. The direction changes are detected in an angular interval equal to a quarter of the angular step of the graduation.

The frequency of the pulses generated by the encoder is proportional to the speed of the rotation. The error of the measurement is inversely proportional to the speed, therefore the procedure is appropriate for high speed region.

At low speeds the measurement of the period of the encoder pulses is recommended. The measurement error is decreasing with the decreasing of the speed.

In case of large speed variations – in order to minimize the errors – a switching between the two described methods is suitable.

The presented simulation structure of the incremental encoder, position and speed computation may be integrated in any Matlab-Simulink® structure.

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