



Experimental Investigation on Robust Control of Induction Motor Using H_∞ Output Feedback Controller

Dénes FODOR

Department of Electrical Engineering and Information Systems,
Faculty of Information Technology
University of Pannonia, Veszprém, Hungary,
e-mail: fodor@almos.vein.hu

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Abstract: This paper deals with H_∞ controller design and real-time experimentation of it for three-phase induction motor control. The model of the motor is given in its state space representation in d-q reference frame. An H_∞ feedback controller is used in the speed loop, which was synthesized by combining a full information controller with an estimator to achieve the desired results. Simplification on the model during the design process has been considered as modelling noise of the system. There are exogenous inputs considered as disturbances, which are not correlated with the measurement noise. The controller was synthesized to minimize the effects of the disturbances entering the plant and the influence of the measurement noise and modelling errors. The desired controller is given by a state space model. The simulation of the system was done in MATLAB/Simulink. The controller is realized as a Simulink embedded S-function. The real-time implementation of the proposed structure has been done on the dSPACE DS1102 DSP development board, with very promising results, showing good reference tracking and good dynamical behaviour.

Keywords: Robust control, induction motor, field-oriented control, estimation technique, implementation issues.

1. Introduction

In the past few years the performance of computers has increased dramatically. With this increased performance it is possible to develop more complex control systems for real life applications. These new control methods are more robust and more reliable than the others, because they can handle complex plant models. The H_∞ theory is a new method and only a few scientific papers with the application to induction motors can be found in the scientific literature [1]. The goal is to demonstrate the practical applicability of the H_∞ controller in an industrial environment. The electrical drive with three phase asynchronous motor is widely used in industry, its main drawback is the

difficult control possibility, so it is ideal for a benchmark application to prove the usefulness of H_∞ control theory.

2. Induction motor model

Table 1 lists the symbols used in this article. Symbols representing vectors are underlined.

Table 1: List of symbols

Meaning	Notation	Meaning	Notation
Control input	$u, u(t)$	Stator current (d,q)	i_{Sd}, i_{Sq}
Disturbances, noise	$w, w(t)$	Rotor speed	ω
Output	$y, y(t)$	Flux speed, angle	ω_{mR}, ε
Measurement	$m, m(t)$	Noise, reference	n, r
Stator voltage (d,q)	u_{Sd}, u_{Sq}	Leakage factor	σ
Magnetizing current	i_{mR}	Rotor, stator time constant	T_R, T_S
Number of pole pairs	p	Inertia	J
Load torque	m_L	Rotor flux vector	$\underline{\Psi}_R$

The dynamic behaviour of the induction motor is described by a set of nonlinear so called general equations. Differential equations (1)-(11) describe the behaviour of the AC motor in the rotor-field-oriented synchronously rotating d-q reference frame [2].

$$\frac{d}{dt} i_{Sd}(t) = \eta_1 i_{Sd}(t) - \eta_2 i_{mR}(t) + \omega_{mR} i_{Sq}(t) + \eta_3 u_{Sd}(t); \quad (1)$$

$$\frac{d}{dt} i_{mR}(t) = \frac{1}{T_R} i_{Sd}(t) - \frac{1}{T_R} i_{mR}(t); \quad (2)$$

$$\frac{d}{dt} i_{Sq}(t) = \eta_4 i_{Sq}(t) - \eta_5 \omega_{mR}(t) i_{mR}(t) - \omega_{mR}(t) i_{Sd}(t) + \eta_3 u_{Sq}(t); \quad (3)$$

$$\frac{d}{dt} \omega(t) = \frac{2}{3} \frac{p^2}{J} (1 - \sigma) L_S i_{mR}(t) i_{Sq}(t) - \frac{z_p}{J} m_L; \quad (4)$$

$$i_{mR}(t) = \frac{1}{L_H} \underline{\Psi}_R(t) e^{j\varepsilon(t)}; \quad (5)$$

$$\omega = \omega_{mR} - \frac{i_{Sq}}{T_R i_{mR}}; \quad (6)$$

$$\eta_1 = -\frac{1}{\sigma T_S} - \frac{(1-\sigma)}{\sigma} \frac{1}{T_R}; \quad (7)$$

$$\eta_2 = -\frac{(1-\sigma)}{\sigma} \frac{1}{T_R}; \quad (8)$$

$$\eta_3 = \frac{1}{\sigma L_S}; \quad (9)$$

$$\eta_4 = -\frac{1}{\sigma T_S}; \quad (10)$$

$$\eta_5 = -\frac{(1-\sigma)}{\sigma}; \quad (11)$$

The above equations result in a nonlinear, time-variant state-space representation. For the controller synthesis let us assume that during normal operation of the drive the modulus of the flux is constant. From (5) it results that $i_{mR} = i_{mR}^{ref}$ is constant, and so $\frac{d}{dt} i_{mR}(t) = 0$ in (2). Then i_{sd} is equal to i_{mR} , and so $\frac{d}{dt} i_{sd}(t) = 0$. (1) is then not a differential equation, but an algebraic one. Substituting (6) into (3) and (4), and using $i_{mR} = i_{mR}^{ref}$, $i_{sd} = i_{mR}$, it results the following simplified state space representation of the AC drive:

$$\frac{d}{dt} \begin{bmatrix} i_{Sq} \\ \omega \end{bmatrix} = A \begin{bmatrix} i_{Sq} \\ \omega \end{bmatrix} + B_u [u_{Sq}] + B_w \begin{bmatrix} m_L \\ n \end{bmatrix}; \quad (12)$$

$$A = \begin{bmatrix} -\frac{1}{\sigma T_S} - \frac{1-\sigma}{\sigma} \frac{1}{T_R} - \frac{1}{T_R} & -\frac{1-\sigma}{\sigma} i_{mR}^{ref} - i_{mR}^{ref} \\ \frac{2}{3} \frac{p^2}{J} (1-\sigma) L_S i_{mR}^{ref} & 0 \end{bmatrix}; \quad (13)$$

$$B_u = \begin{bmatrix} \frac{1}{\sigma L_S} \\ 0 \end{bmatrix}; \quad (14)$$

$$B_w = \begin{bmatrix} 0 & N_1 \\ -\frac{p}{J} & N_2 \end{bmatrix}; \quad (15)$$

The load torque is assumed to be a disturbance, and n represents the disturbances resulting from the imprecise definition of the reference value i_{mR}^{ref} , cross effects between the state variables, the general system noise and the unmodelled dynamics of the system. Coefficients are bounded and can be determined by taking the upper bound of the disturbances modulus. The model presented in (12) is the nominal plant, which will be used for controller design. The controller will be used for the original model (1)-(11), that is the perturbed plant presented in *Fig. 1*.

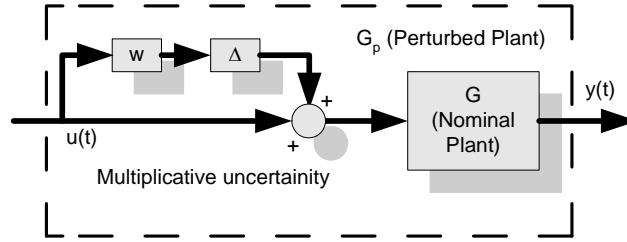


Figure 1: Model used in controller design.

The only input of the model (12) is the q component of the stator voltage.

3. The H_∞ controller design

The goal is to design an H_∞ controller for the three-phase asynchronous motor in the speed loop. The assumption is that there is noise entering the plant, so the estimator part of the controller was used, too. The plant is described in general case by the following equations [3], [4]:

$$\dot{x}(t) = Ax(t) + \begin{bmatrix} B_u & B_w \end{bmatrix} \begin{bmatrix} u(t) \\ w(t) \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} m(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_m \\ C_y \end{bmatrix} x(t) + \begin{bmatrix} 0 & D_{mw} \\ D_{yu} & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ w(t) \end{bmatrix} \quad (17)$$

The following conditions need to be satisfied [5]:

- $D_{mw}B_w^T = 0$;
- $D_{mw}D_{mw}^T = I$;
- $D_{yu}^T C_y = 0$;
- $D_{yu}^T D_{yu} = I$;

- The plant is controllable from the control input and from the disturbance input;
- The plant is observable from the measured output and from the reference output.

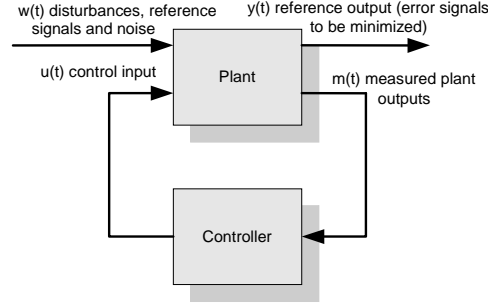


Figure 2: General control configuration.

If these requirements are satisfied, a controller can be designed according to the general control configuration in Fig. 2. The internal structure of the controller is shown in Fig. 3. The estimator part estimates the state and the feedback part generates the control input. The solution of the finite-time steady state suboptimal H_∞ output control problem can be given by solving two Riccati equations. The suboptimal controller is defined according to (18), the matrices can be calculated using the solutions of the algebraic Riccati equations [3].

$$\begin{aligned} \dot{x}_c(t) &= A_c(t)x_c(t) + B_c(t)m(t) \\ u(t) &= C_c(t)x_c(t) \end{aligned} \quad (18)$$

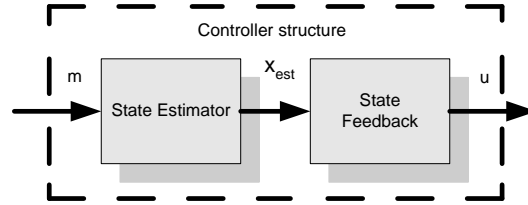


Figure 3: Structure of the controller.

4. Adaptation of the motor model to the H_∞ control method

The model presented in (12) is appropriate for the control synthesis. To guarantee good reference tracking, the speed-reference input is considered as a disturbance input and all coefficients of this input in the equation (12) are set to zero, since the reference input has no direct influence on the equations of the motor. The output equations need to be formulated to have a representation as in

(16) and (17). Note that, since all parameters are fixed, the description is time-invariant. The measured value is the difference between the reference speed and the actual speed together with the measurement noise. The reference output consists of the difference between reference and actual speed (without noise), i_{Sq} , and u_{Sq} . The controller should keep the control input finite and should not try to set it to zero, so the weights for the stator current and voltage in the reference output are kept small. Note that the disturbance input is included in the reference output, which is not consistent with the H_∞ problem statement as in (16) and (17). This drawback can be solved by applying more general suboptimal control formulas [6]. In order to make calculations easier, a steady state gain was used to design the controller. Some performance limitations have to be made [5] to guarantee good reference tracking (after the design ad hoc integral action was added [3]) and the output signals were bounded. The final model of the system for controller design results:

$$\frac{d}{dt} \begin{bmatrix} i_{Sq} \\ \omega \end{bmatrix} = A \begin{bmatrix} i_{Sq} \\ \omega \end{bmatrix} + B_u [u_{Sq}] + B_{wf} \begin{bmatrix} m_L \\ n \\ r \end{bmatrix}; \quad (19)$$

$$\begin{bmatrix} m(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ 0.01 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_{Sq} \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{Sq} \\ m_L \\ n \\ r \end{bmatrix}; \quad (20)$$

$$B_{wf} = \begin{bmatrix} 0 & N_1 & 0 \\ -\frac{p}{J} & N_2 & 0 \end{bmatrix}. \quad (21)$$

The controller has the form (18). After solving the Riccati equations, the matrices of the controller A_c , B_c , C_c can be calculated using the solutions of the equations and the state matrices of the model. A performance bound required for the solution was chosen approximately 10% over theoretical optimum.

5. Simulation results

Parameters of the induction drive used in the simulation:

$$L_s = 0.13 \text{ H}, L_h = 0.12 \text{ H}, L_r = 0.13 \text{ H}, R_r = 3.0 \Omega, R_s = 1.86 \Omega, p = 2.$$

The simulation was made with MATLAB/Simulink, according to the structure presented in *Fig. 4*. The model representing the AC drive is a time-

variant model described by the equations (1)-(11), and not the simplistic model used for controller design. The inputs of the motor are three phase stator voltages and the load torque. The controller provides the value of u_{sq} , while u_{sd} is kept constant. Flux computation, presented in *Fig. 5*, was used in order to get the angle of the rotor flux, which would allow the transformation of the voltages from d-q reference with reverse Park- and reverse Clarke-transformation into the a, b, c three-phase components.

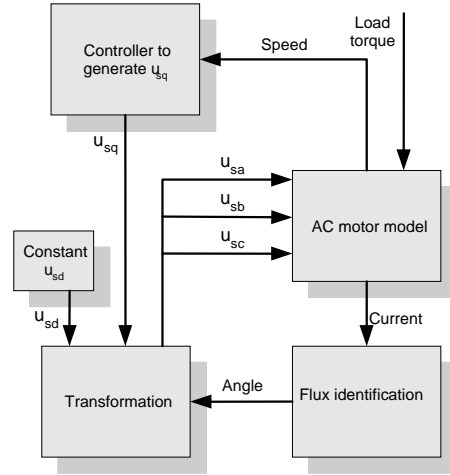


Figure 4: The structure of the drive model.

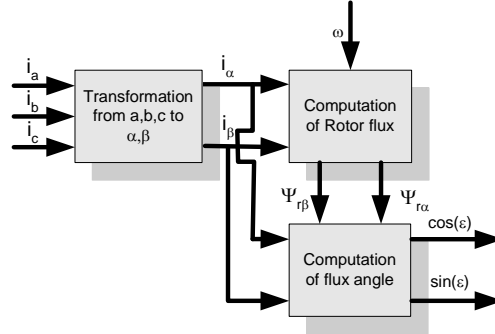


Figure 5: Flux identification used in the simulation.

The simulation results are presented in *Fig. 6-8*. *Fig. 6* shows the reference tracking capability, even with large measurement noise. The load torque is presented in *Fig. 7*. The simulation results show that the H_∞ controller works well, and the system with rapidly changing reference shows good dynamic behaviour.

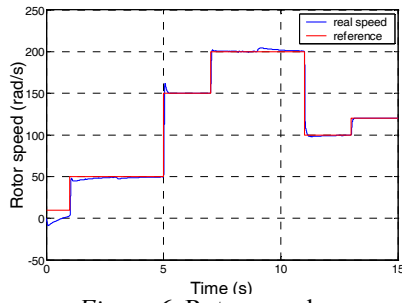


Figure 6: Rotor speed.

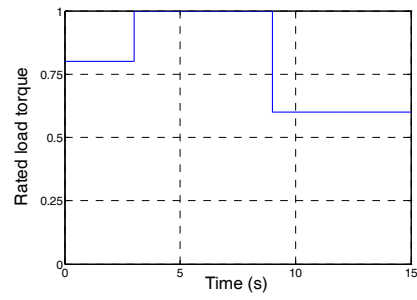
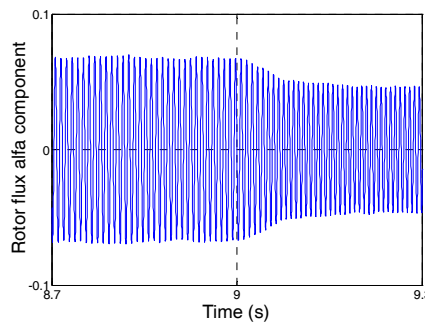


Figure 7: Load torque.

Figure 8: Rotor flux d component.

From the simulation results presented in *Fig. 9-10*, it is clear that the controller shows good results even when load torque changes rapidly. Note, that during controller design, the load was considered as disturbance; according to (4), it influences the rotor speed more than the measurement noise. Thanks to the robustness of the H_∞ controller, the measurement errors and modeling errors are compensated.

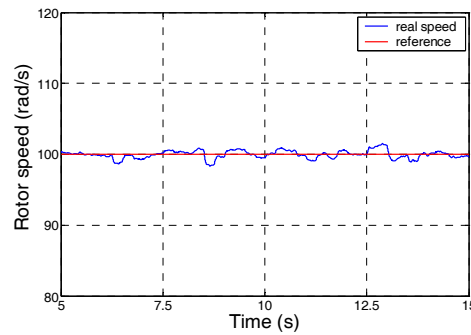


Figure 9: Reference tracking (100 rad/s) during rapid variation of the load.

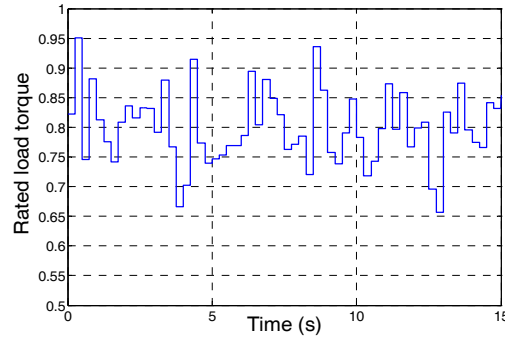


Figure 10: Rapidly changing load.

6. Testing the dynamic behaviour of the induction motor

With the experimental setup based on the dSPACE DS1102 development board, it is possible to study the dynamic behaviour of the motor in real-time. Note, that the motor model is tested without controller; supplied by three sine-wave generators, with amplitude and frequency values programmable via a graphical user interface. The value of the load torque can be also arbitrarily changed (see Fig. 11). The values of the rotor flux, stator current and speed are displayed on plotters. Robustness tests can be made by changing the position of the slider bar indicating the value of the rotor resistance. By changing this value, the parameters of the transfer functions are also changed. These are displayed numerically on the right.

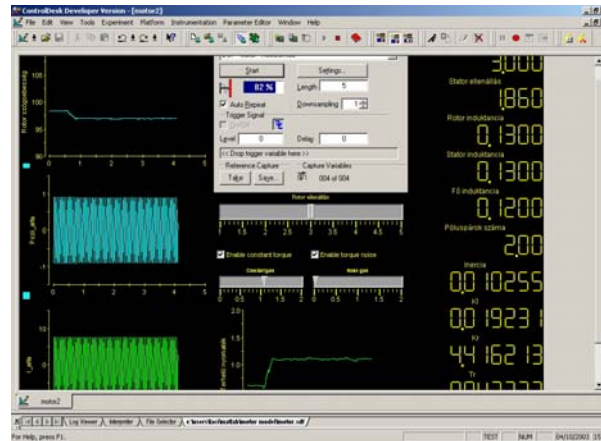


Figure 11: Test of the motor model.

7. Experimental results on the DS1102

In the second experiment, the motor model is tested with the controller whose design was presented in section 3 of the paper. *Fig. 13* shows the measured values when executing the real-time code. There are four plotters on the screen. The top-left shows the reference and actual speed values (it can be said that the controller shows good tracking capabilities). The other displays show the value of the rotor flux, the tracking error and the load torque. There are two slide bars at the bottom. The left slide bar enables the user to set the value of the load torque during code execution. The right slide bar can be used to set the magnetic operating point (the desired flux value) and thus test how it affects the control process. With these and other similar GUI-s functionalities, all aspects and effects of parameter changes can be studied in real-time. The functionality of the system will be shortly discussed as follows.

The ds1102 board (with TMS320C31 processor) is a PC card designed for development of high-speed multivariable digital controllers equipped with analogue to digital and digital to analogue converters. ControlDesk software [7] provides a framework to manage the DSP board [8]. It has a real-time interface (RTI) to MATLAB/Simulink and additional blocksets as well. With this interface, real-time code can be easily built, downloaded and executed. Instrument panels (also called layouts, presented later) can be easily made using ControlDesk, allowing the change of parameters and real-time display of all state-variables and signals in the system. The main advantage of the ControlDesk is that the code -executable on the DSP- can be directly generated from the Simulink model of the system. This can be done in the following way. The model of the system is built in Simulink using the original Simulink blocks (and using only the blocks which are fully supported by ControlDesk), and blocks handling the board's hardware such as in- and outputs, interrupts. As it was mentioned before, there are blocks, which can not be used with RTI. These blocks must be replaced by combinations of other blocks before the code is generated. Using RTI, several different settings are possible. The generated code can be single- or multitasking, block reduction can be switched on or off, signals can be reused, and so on. The best results can be achieved when all parameters are *tuneable*.

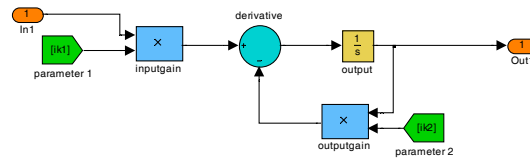


Figure 12: Block diagram of the transfer function.

In order to do this, it is necessary to replace transfer functions with parameter coefficients (such as $1/(Ts+1)$ for example) with a combination of blocks from integrators and gain blocks (see Fig. 12). To be able to change the parameters on-line, the parameters have to be masked (or else they are not accessible from ControlDesk during code execution). By changing parameters, it becomes possible to make robustness tests (by changing the induction motor parameters and thus simulating parametric uncertainty), as well as to tune the controller and to give different input signals to the systems input. With the user interface, parameters can be easily tuned, values displayed or saved. First, the controller is simulated with the mathematical model of the system built in Simulink. After having good results, the blocks, representing the theoretical model of the induction motor, are replaced by blocks representing the real induction motor (such blocks as inputs and outputs, interrupt handler blocks and so on). After the blocks are replaced, the system can be tested with the real induction motor.

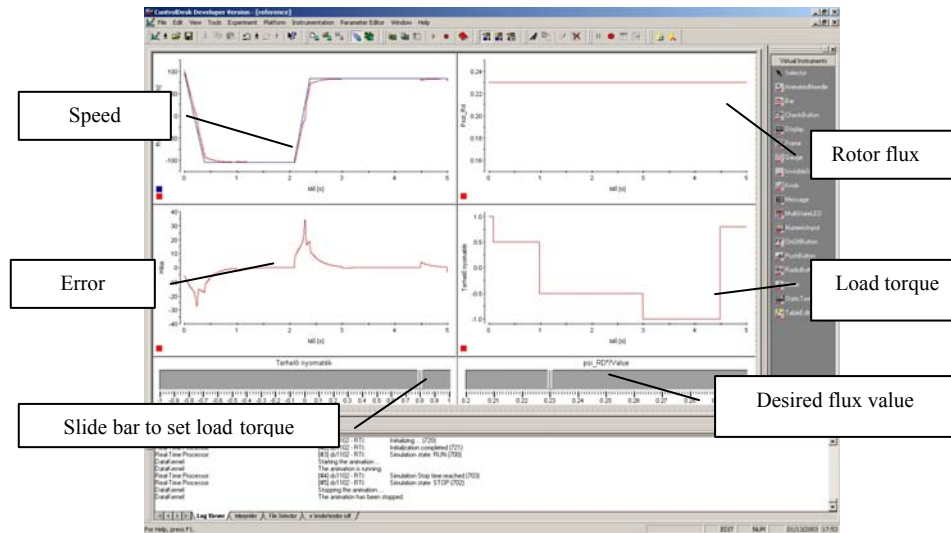


Figure 13: Experimental results displayed during real-time code execution.

8. Conclusion

According to the theoretical background presented in section 2 and 3, an H_∞ controller was designed by making some simplifications on the motor model and then the controller's behaviour was tested with it. It was shown that all assumptions, simplifications made during the design process are valid. The system showed good dynamic behaviour and it was shown how the performance (noise rejection, robustness, and tracking) depends on the design parameters. In

consequence, the controller can be implemented for real-time usage as the last sections show. The controller can be further developed by taking into account more disturbances. Remark that actuator errors (resulting from the switching behaviour of the inverter) and modelling noise were not used in the controller design, and current measurements were assumed to be precise (it was assumed to measure them without noise). The performance can be further improved by using a flux model, which is further developed so that it gives more precise values during transient processes, or by using a speed estimator [9]. Since the motor model is nonlinear, a parameter-variant or nonlinear controller can be taken into account as well [10].

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