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Adapted Discrete Wavelet Function Design for ECG Signal Analysis

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Abstract: The main task in wavelet analysis (decomposition and reconstruction) is to find a good wavelet function (mother wavelet) to perform an optimal decomposition. The goal of most wavelet researches is to create a set of basis functions and transforms that will give an informative, efficient and useful description of a signal. It is better if the wavelet function is adapted to the signal, because the computational costs can be reduced and more accurate analysis can be obtained. This paper presents a discrete wavelet function synthesizer, which starts from an arbitrary, discretized sequence, to obtain the reconstruction and decomposition filters. The pursued criterion (expected result) is to minimize the reconstruction error between a first or second order approximation and the original signal.

Keywords: wavelet analysis, decomposition and reconstruction filter banks, multiresolution analysis, discrete wavelet transform

1. Introduction

The analysis of an ECG signal has been used as a diagnostic tool to provide information on the functions of the heart. The ECG is the graphical representation of variation in time of a potential difference between two points on a human body surface as a result of the activity of the heart. The wavelet transform is a recently developed signal processing technique, created to overcome the limits of the classical Fourier analysis, to deal with non-stationary signals like biomedical signals. The wavelet transform of a signal is calculated by taking the convolutive product between the biological signal and basis functions, measuring the similarity between them. The result of this product is a set of coefficients. This set of coefficients indicates how similar is the signal relative to the basis functions. In the case of wavelet analysis, the basis functions are scaled (stretched or compressed) and translated versions of the same prototype function, called the mother wavelet $\psi(t)$. Theoretical knowledge about mathematical backgrounds of wavelet transform can be found in [1], [2], [3]. This paper briefly introduces a new method, using softcomputing elements to synthesize new wavelet functions in order to have with them an optimal decomposition structure. The expected result is to minimize the reconstruction error between a first order approximation and the original signal.

2. Wavelet Decomposition and Reconstruction

The wavelet transform is a decomposition of the signal as a combination of a set of basis functions, obtained by means of scaling a and translation b of a mother wavelet $\psi(t)$. The continuous wavelet transform (CWT) uses the dilation and translation of the mother wavelet function ψ. The CWT of signal $x(t)$ is defined as [1]:

$$W_a x(b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} x(t) \cdot \overline{\psi}\left(\frac{t-b}{a}\right) dt \,. \tag{1}$$

where a is a scale factor which is proportional to the inverse of frequency and b is the translation parameter. The scale factor and the translation parameter can be discretized, the usual choice is to follow a dyadic grid for them. The transform is then called the (dyadic) discrete wavelet transform (DWT).

$$C(j,k) = \sum_{n \in Z} x(n) \cdot \psi_{j,k}(n), \; \psi_{j,k}(n) = 2^{-j/2} \cdot \psi\left(2^{-j} n - k\right) \tag{2}$$

For discrete time-signals, the dyadic discrete wavelet transform (DWT) is equivalent according to Mallat’s algorithm [1] to an octave filter bank, and can be implemented as a cascade of identical filter cells (low-pass and high-pass finite impulse response(FIR) filters) as shown in *Fig. 1*.

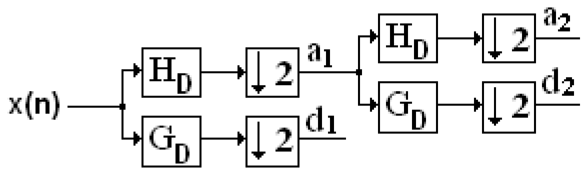

Figure1: FIR filter structure for dyadic scale decomposition.

The decomposition procedure starts with passing signal (discrete sequence) through a half band digital low-pass filter H_D with impulse response h(n). Filtering corresponds to the convolution of signal with the impulse response of the filter. A half band low-pass filter removes all frequencies that are above half of the highest frequency in the signal, but leaves the scale unchanged. Only the subsampling process changes the scale. In summary the low-pass filtering halves the resolution but leaves the scale unchanged. The signal is then subsampled by 2 since half of the number of samples is redundant. This operation doubles the scale (*Fig.1*). The operators H_D and G_D correspond to one stage in the wavelet decomposition, the spectrum of the signal is split in two equal parts, a low-pass (smoothed) and the high-pass part. The low-pass part can be split again and again until the number of bands created satisfies the computational demands. Thus, the discrete wavelet transformation can be summarized (after *j* stages) as

$$x \rightarrow \left(Gx, GHx, GH^2x, ...GH^{j-1}x, H^j x\right) \rightarrow \left(d_{j-1}, d_{j-2}, ...d_1, a_1\right) \qquad (3)$$

The output of the DWT consists of the remaining several times smoothed components, and all of the accumulated "detail" components [5].
The reconstruction procedure is similar to decomposition. The signal at every level is upsampled by two, passed through the synthesis (low-pass and high-pass) filters and then the filtered components are summed [4].

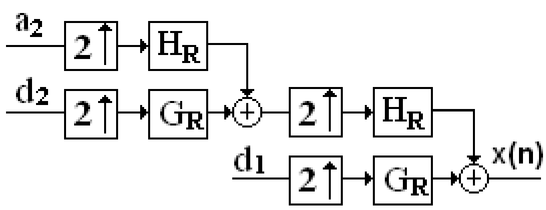

Figure 2: FIR filter structure for dyadic scale reconstruction.

The filters must satisfy certain requirements to enable perfect reconstruction from the two output signals after downsampling, and to yield an orthogonal underlying wavelet basis. To end up with a corresponding mother wavelet $\psi(t)$ having compact support, the filters (H_D,G_D,H_R,G_R) must be finite impulse response (FIR) filters [5]. All the filters are intimately related to the sequence $W = (w_n), n \in Z$ which defines the dilation (or refinement) relation

$$\frac{1}{2}\phi\left(\frac{x}{2}\right) = \sum_{n\in Z} w_n \phi(x-n). \qquad (4)$$

If ϕ is compactly supported, the sequence $W = (w_n), n \in Z$ can be viewed as a filter, and from this we can define four FIR filters of length 2N organized as in *Fig. 3*. G_R and H_R are quadrature mirror filters (qmf) [3], H_D is obtained from H_R by flipping its coefficients. H_D and G_D are also quadrature mirror filters [3].

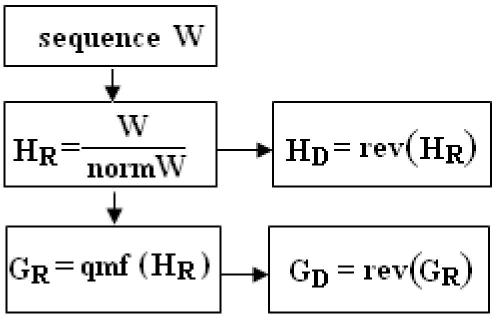

Figure 3: FIR filter synthesis according to the perfect reconstruction conditions.

3. Method and materials

The wavelet functions are obtained using an artificial neural network based function synthesizer. The basis function (wavelet) sequence is synthesized following the algorithm presented in *Fig. 4*. The main criteria for these filters are: low-pass FIR filter of *2N* length, with norm $\sqrt{2}$; the low-pass and high-pass structures are obtained from this arbitrary sequence.

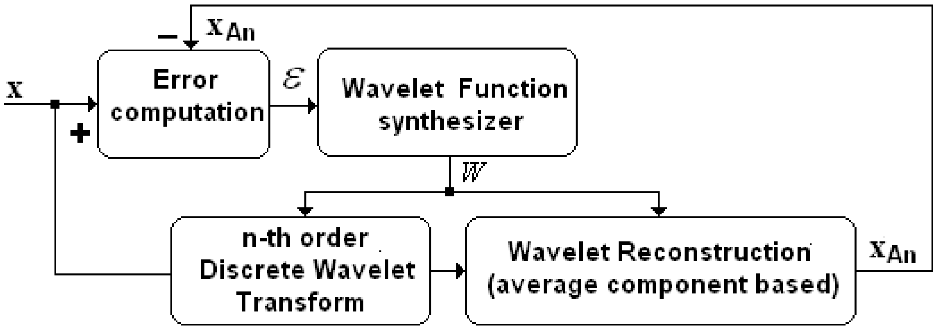

Figure 4: Function synthesizer.

The calculation of the error ε is performed comparing the reconstructed function (from first average components) with the signal. The approximation error is defined as a difference between the signal x and the function reconstructed from x_{An} average components only.

$$\varepsilon = \varepsilon_{wavelet} + \varepsilon_{norm} \tag{6}$$

$$\varepsilon_{wavelet} = \{X - IDWT[DWT(X)]\}^2 \tag{7}$$

$$\varepsilon_{norm} = \left(norm(W) - \sqrt{2}\right)^2 \tag{8}$$

The criterion-function was defined as:

$$w_i = w_i + \mu \frac{\delta\varepsilon}{\delta w_i}, \tag{9}$$

where μ is the learning rate and $\frac{\delta\varepsilon}{\delta w_i}$ is the variation of error. The used test signal is from MIT-BIH Arrhythmia Database.

4. Results

The test signal is from MIT-BIH database; we used only a short sequence and several existing wavelet functions. The resulted wavelet sequence is presented in *Fig. 4*, the analyzed signal and its average and detail components are presented in *Fig. 5*.

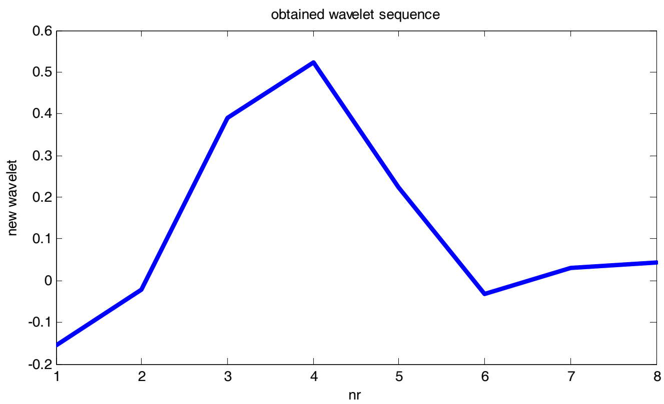

Figure 4: The resulted wavelet function.

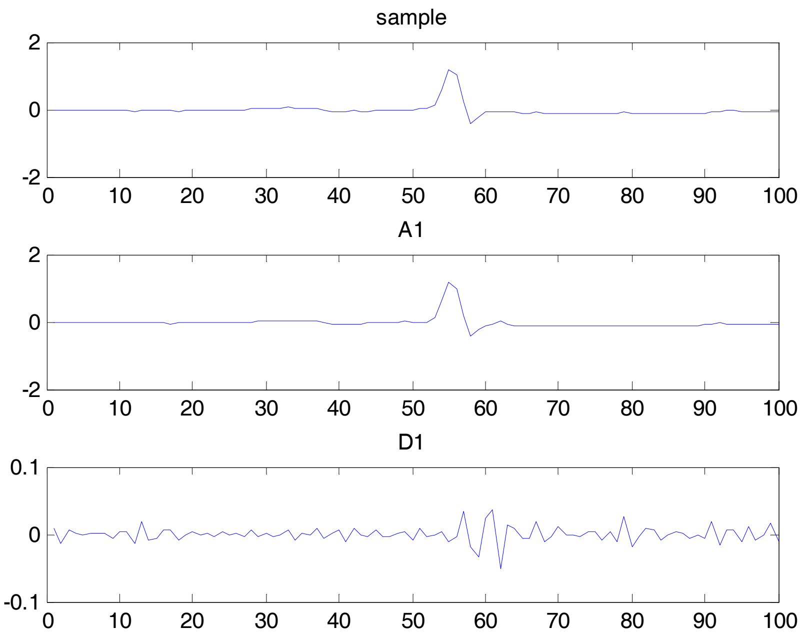

Figure 5: The analyzed signal sequence and its first average and detail components.

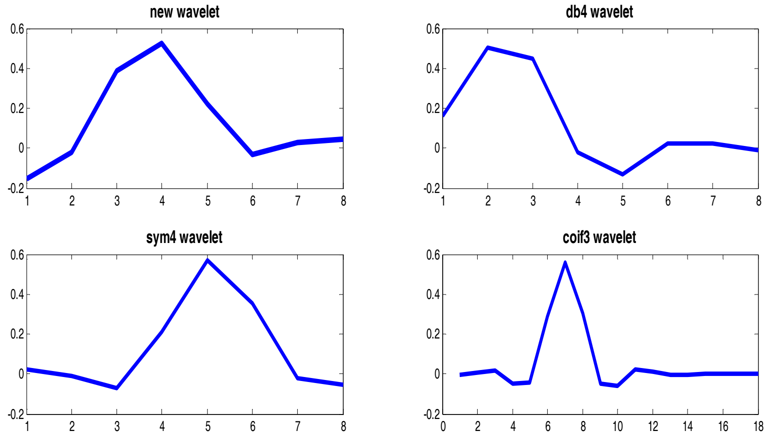

Figure 6: The obtained wavelet sequence compared to other similar functions

5. Conclusions

The main advantage of the presented method is that the analyzing discrete wavelet can be adapted to the signal. Using filter banks it's possible to obtain a discrete wavelet transform of a sequence without specifying any function. The obtained functions gave better or almost the same results in decomposition, reconstruction as the existing functions. Using adapted wavelet functions reduces the computational costs and gives a more accurate multiresolution analysis.

References

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